

# A self-consistent numerical model of the Shear-Layer Oscillation for massive stars.

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## Abstract

Decades of observations of the Earth's equatorial stratosphere reveal a regular oscillation where winds alternate between eastern and western directions. Similar phenomena have also been detected in the atmospheres of other planets such as Jupiter or Saturn. While the same situation occurs in massive stars where a stably-stratified layer (the radiative zone) lies on top of a convectively unstable one (the convective zone), no observations have yet been able to report it, raising question about its existence in a stellar context.

It is believed that internal gravity waves (IGW), excited by an underlying convective region – the troposphere in the case of the Earth – can propagate in the stably stratified layer and lead to this large scale oscillation through angular momentum deposition. While reduced models of this mechanism have been proposed in the past to investigate this behaviour, in which the action of the waves is taken into account through nonlinear closure models, very few studies have investigated the interaction between convective and stable media due to the very constraining time scale separation. In this work, I present global simulations in polar geometry where both regions are simulated without resulting to any such parameterisation. Through a systemic parametric study, I discuss how typical properties of the flows generated in the radiative zone evolve, such as their period or regularity. I conclude by giving estimates of the period of this oscillation for massive stars, discussing future models.

## 1. Numerical setup

Incompressible (Boussinesq) with internal heating  $\tilde{Q}(\tilde{r})$ , Dedalus code [1],

$$\frac{\partial \tilde{\mathbf{u}}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\frac{\nabla \tilde{p}}{\tilde{\rho}_0} + \tilde{\nu} \Delta \tilde{\mathbf{u}} - \frac{\delta \tilde{\rho}}{\tilde{\rho}_0} (\tilde{T}) \tilde{g}(\tilde{r}) \mathbf{e}_r, \quad (1)$$

$$\frac{\partial \tilde{T}}{\partial t} + \tilde{\mathbf{u}} \cdot \nabla \tilde{T} = \tilde{\kappa} \Delta \tilde{T} + \tilde{Q}(\tilde{r}), \quad (2)$$

$$\nabla \cdot \tilde{\mathbf{u}} = 0. \quad (3)$$

Piecewise linear density [2,3],

$$\frac{\delta \tilde{\rho}}{\tilde{\rho}_0}(\tilde{T}) = \begin{cases} S \alpha_{CZ} \tilde{T}, & \tilde{T} \leq \tilde{T}_i, \\ -\alpha_{CZ} \tilde{T}, & \tilde{T} \geq \tilde{T}_i. \end{cases} \quad (4)$$

Control parameters

$$Ra = \frac{\alpha_{CZ} \tilde{g}_0 \tilde{T}_0 \tilde{r}_0^3}{\tilde{\nu} \tilde{\kappa}}, \quad Pr = \frac{\tilde{\nu}}{\tilde{\kappa}}, \quad S, \quad T_o, \quad (5)$$

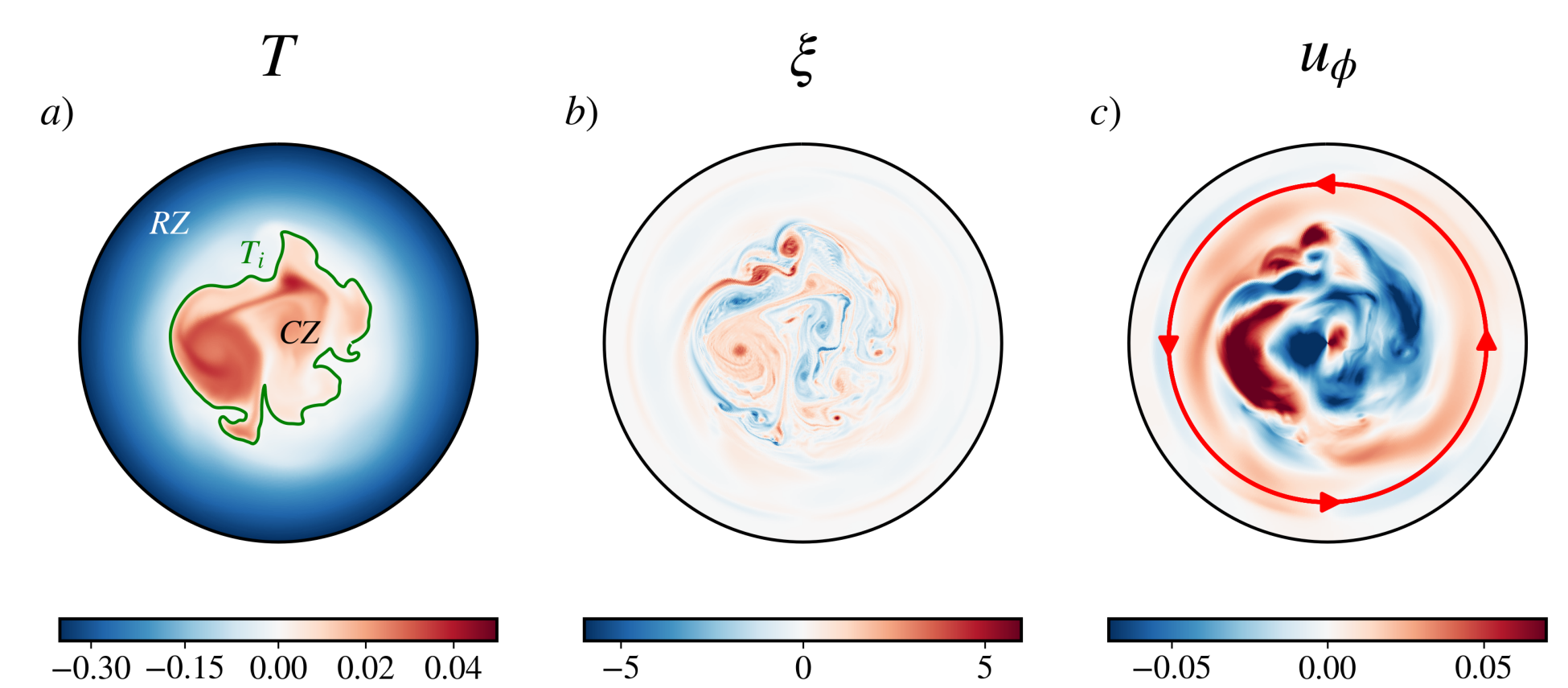
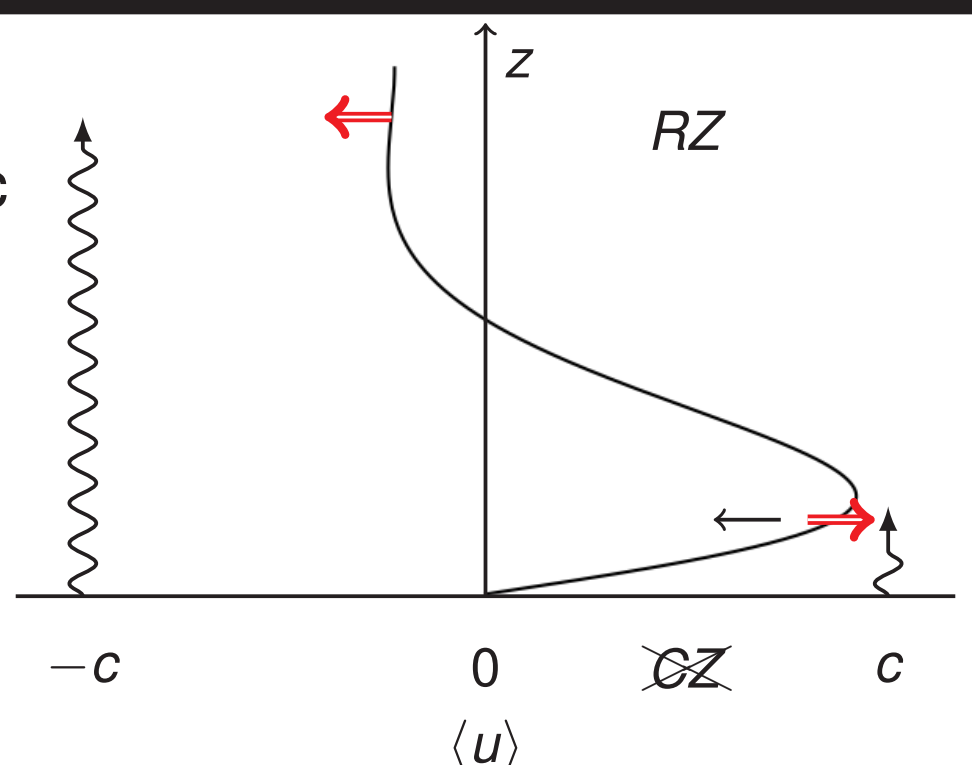


FIGURE 1 – a) Temperature field in a simulation, displaying an active core compared to the envelope. The contour value  $T_i = 0$  denotes the boundary between the convective (CZ) and radiative (RZ) zones. b) Vorticity  $\xi = \nabla \times \mathbf{u}$  of the flow, illustrating the typical turbulence generated in the core of the domain. c) Azimuthal flow developing in the stably-stratified layer whose reversal is at the core of the present study.

## 2. Theoretical considerations

- 1D model:  $\langle u \rangle(z, t)$
- ‘Fast’ monochromatic waves
- ‘Slow’ oscillation if

$$\Lambda_1 = \frac{T_{osc}}{T_{diss}} < cst$$



Plumb and McEwan 1978 [4]

$$\frac{\partial \langle u \rangle}{\partial t} = -\frac{\partial \langle u'w' \rangle}{\partial z} + \Lambda_1 \frac{\partial^2 \langle u \rangle}{\partial z^2}. \quad (6)$$

$$\langle u'w' \rangle = \pm \exp\left(-\int_0^z \frac{dz'}{\langle u \rangle \mp 1}\right)^4. \quad (7)$$

Question:

- How does the problem behave in a star when considering a realistic continuous spectrum of IGW, such as one generated by convection?

## 3. Spectra in the RZ

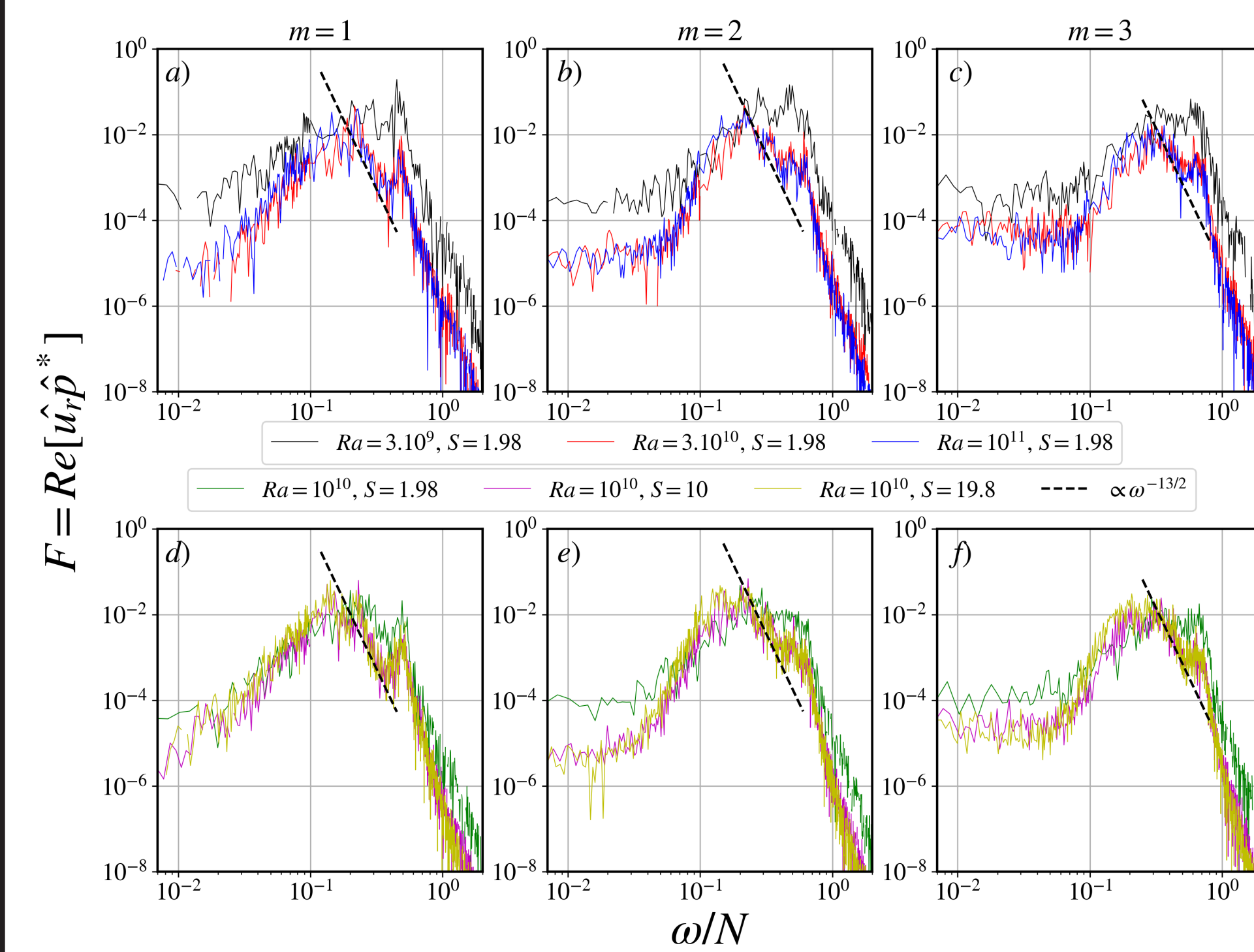


FIGURE 2 – Wave energy flux temporal spectra measurements for varying  $Ra$ , fixed  $S$  (a-c) and fixed  $Ra$ , varying  $S$  (d-f). Spectra are computed in the RZ ( $r = 0.7$ ), and displayed for 3 values of horizontal wavenumbers. The dashed lines show comparisons with [5].

The wave energy flux can be related to the Reynolds stress [6].

## 4. Space and time visualisations

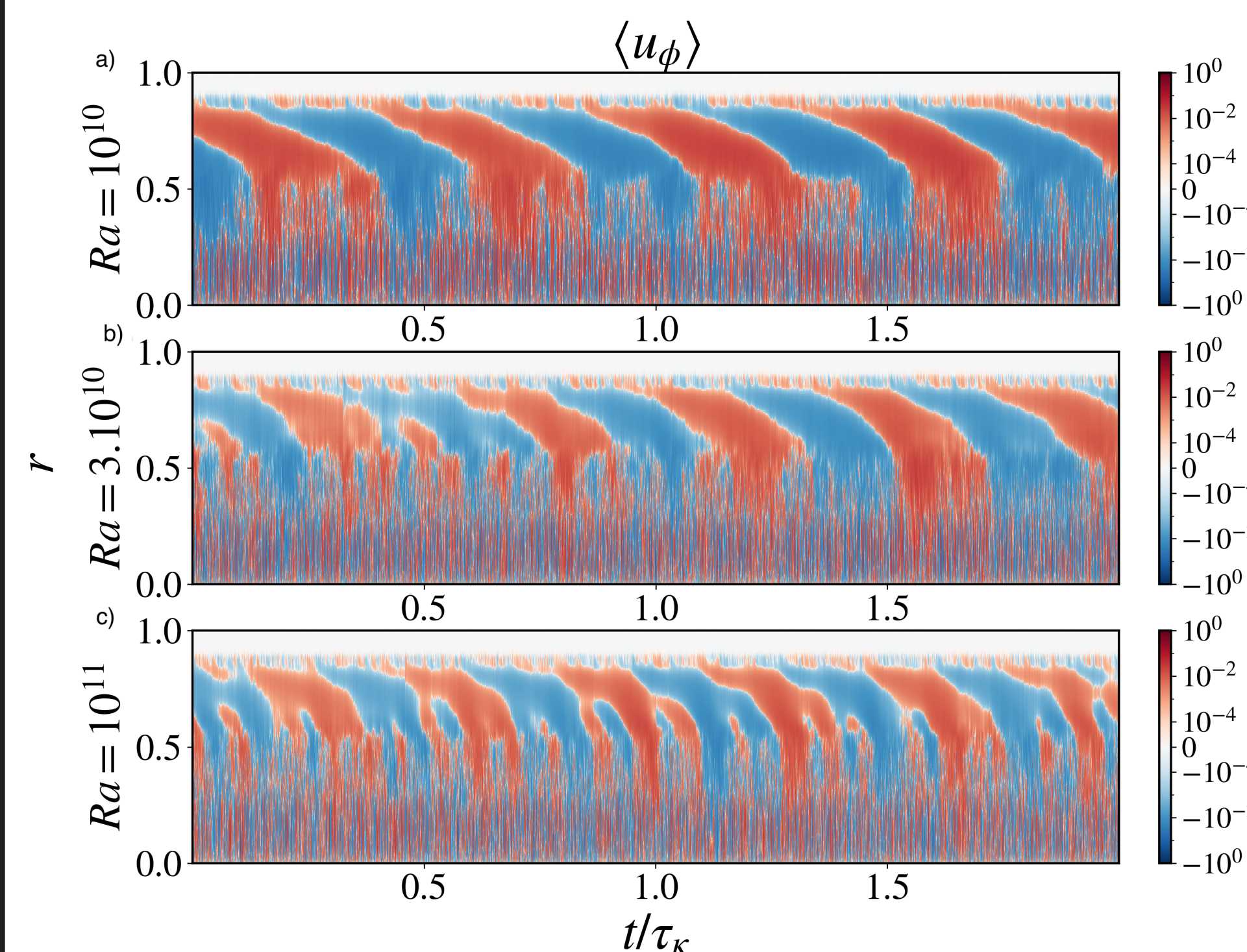


FIGURE 3 – Mean flow visualisation for  $S = 1.98$ . Hovmöller diagrams (colours correspond to flow amplitudes) for  $Ra = 10^{10}$  (a),  $Ra = 3.10^{10}$  (b) and  $Ra = 10^{11}$  (c).

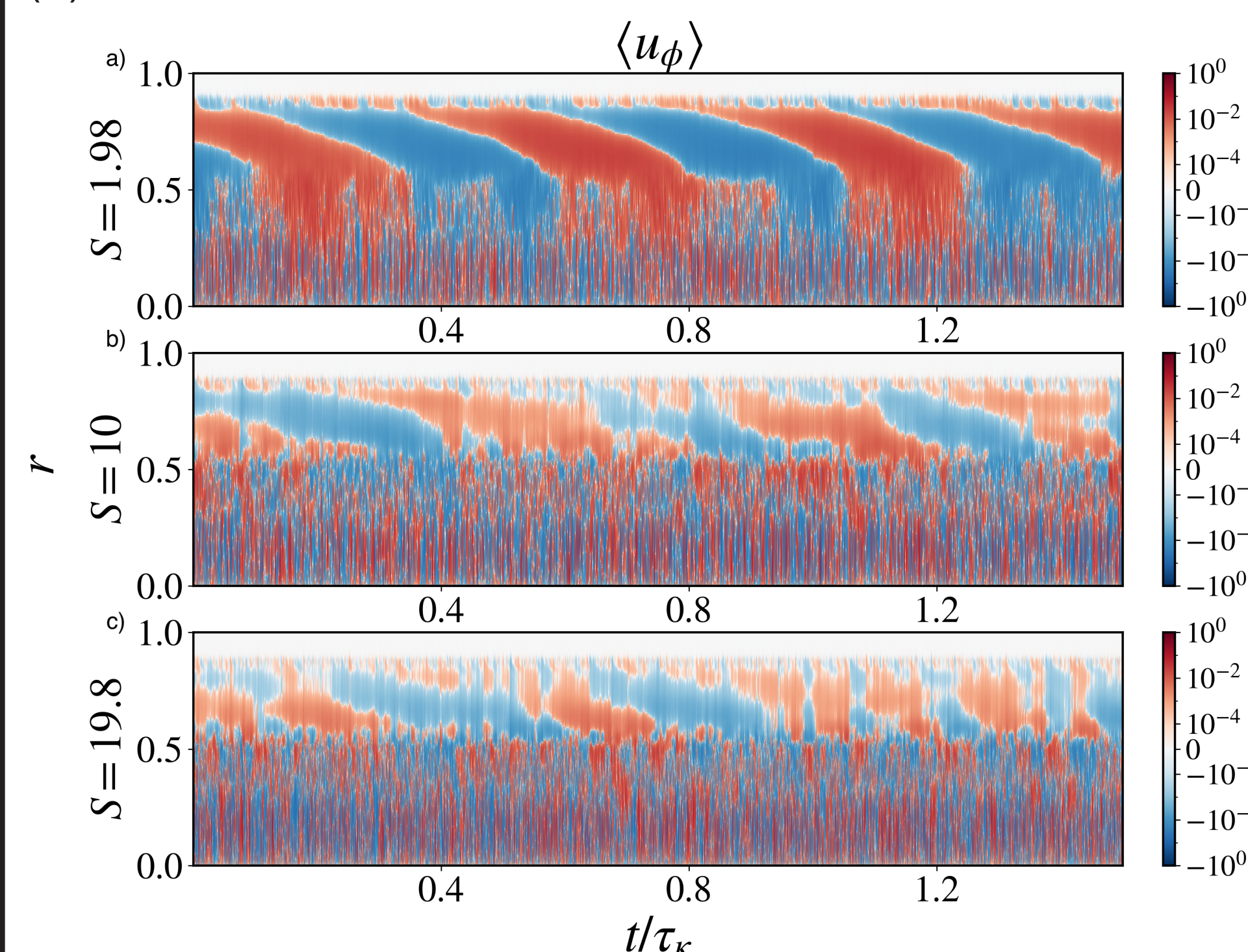


FIGURE 4 – Same as in Figure 3 but for  $Ra = 10^{10}$ ,  $S = 1.98$  (a),  $S = 10$  (b) and  $S = 19.8$  (c).

When the forcing  $Ra$  is high enough, reversals of the mean azimuthal flow occur in the RZ, showing a very slow temporal behaviour compared to the fluctuations in the CZ.

## 5. $\Lambda_1$ in DNS

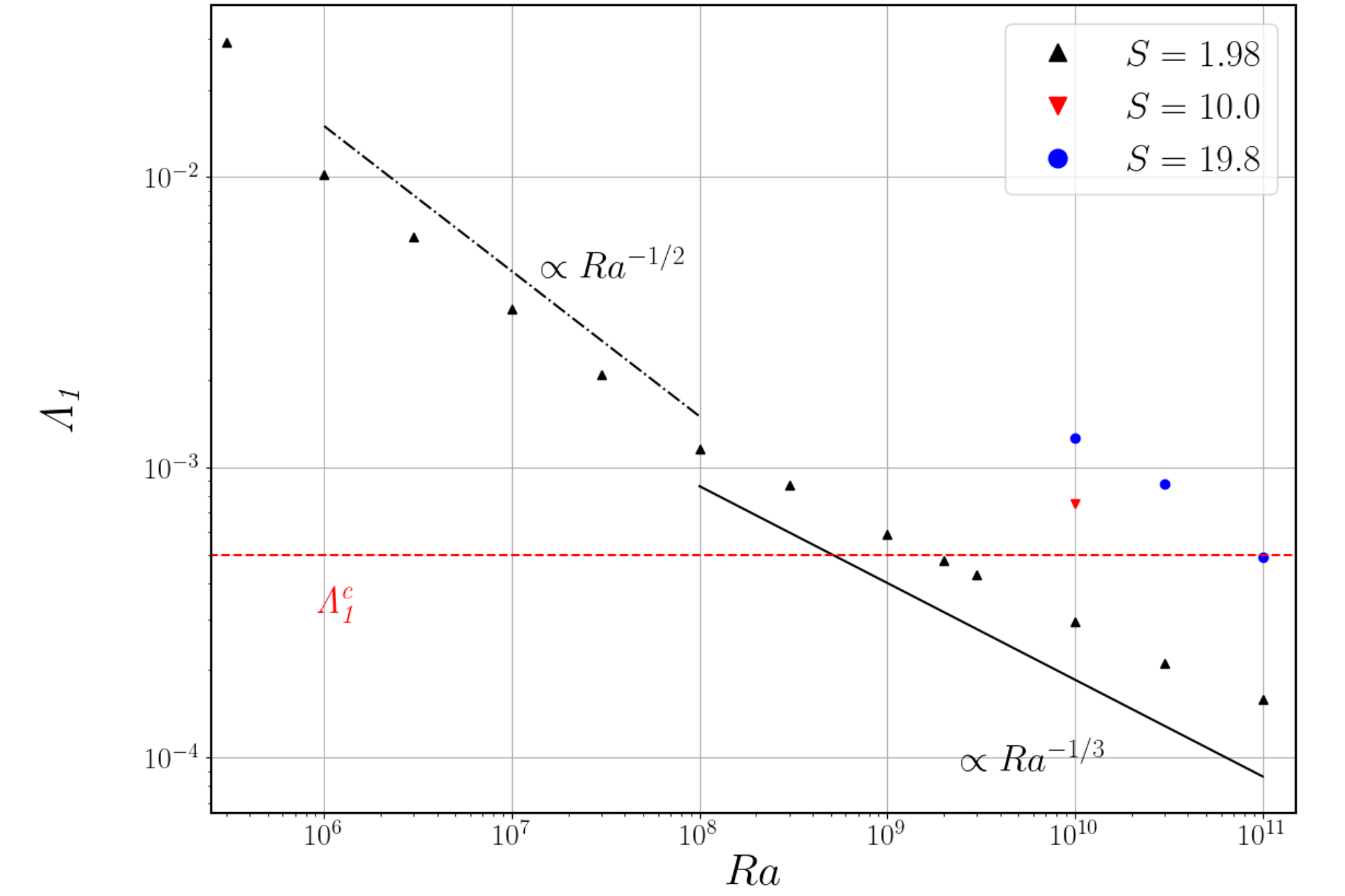


FIGURE 5 – Estimate of  $\Lambda_1 = \frac{\tilde{\nu} c}{d \langle u'_r u'_\phi \rangle}$  for our data points, showing how increasing  $Ra$  tends to favour low  $\Lambda_1$  values and therefore mean flow reversals when  $\Lambda_1 < \Lambda_1^c = 5.10^{-4}$ .

## 6. Prediction for the period

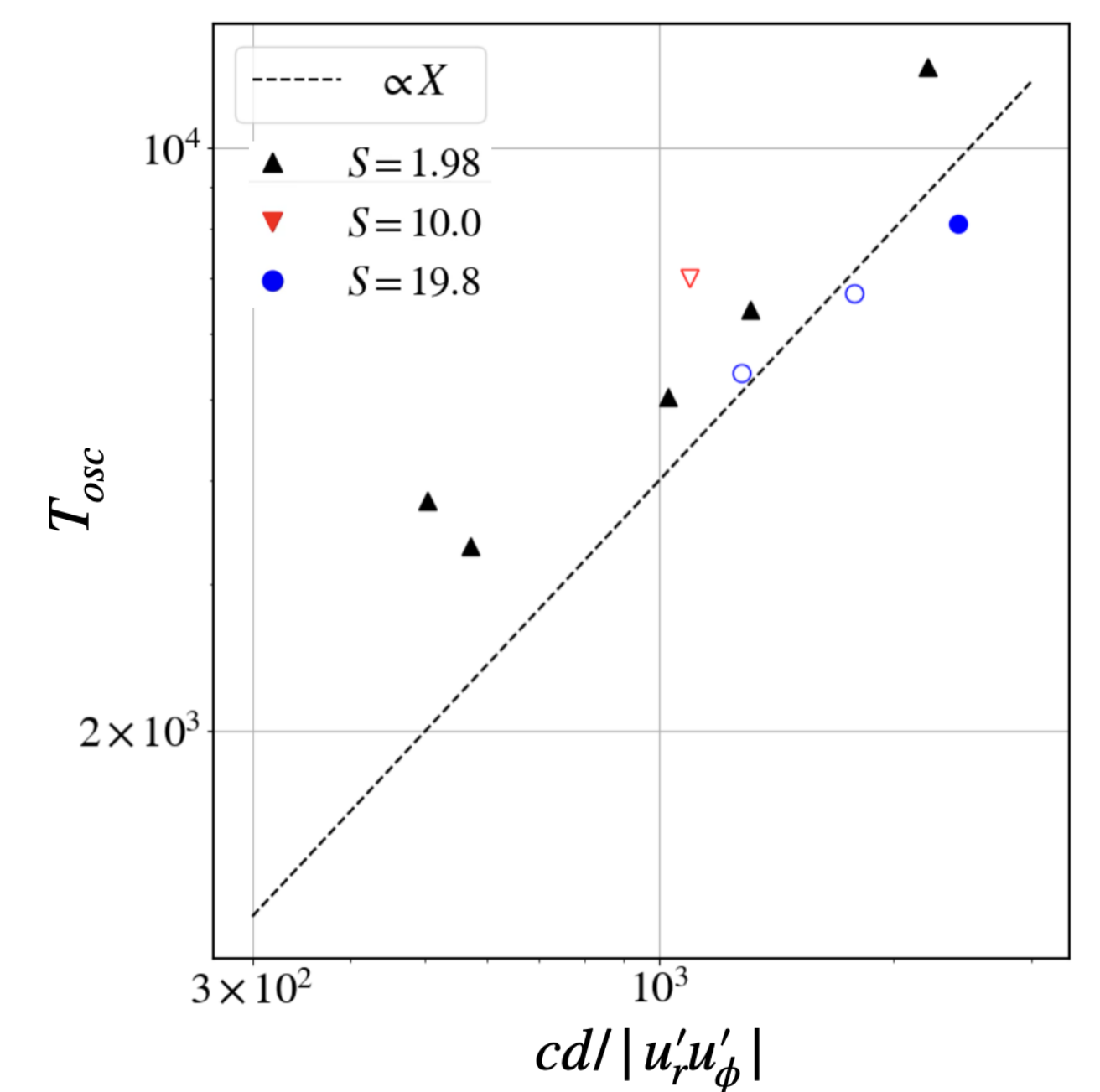


FIGURE 6 – Measure of  $T$  as a function of  $cd / |u'_r u'_\phi|$ .

Prediction for a 15 solar mass star: 5 My.

## Conclusion and future work

- Self-consistent coupling between RZ and CZ in polar geometry
- Missing physical ingredients: rotation, wave-breaking, magnetic fields..

Effects of rotation ?

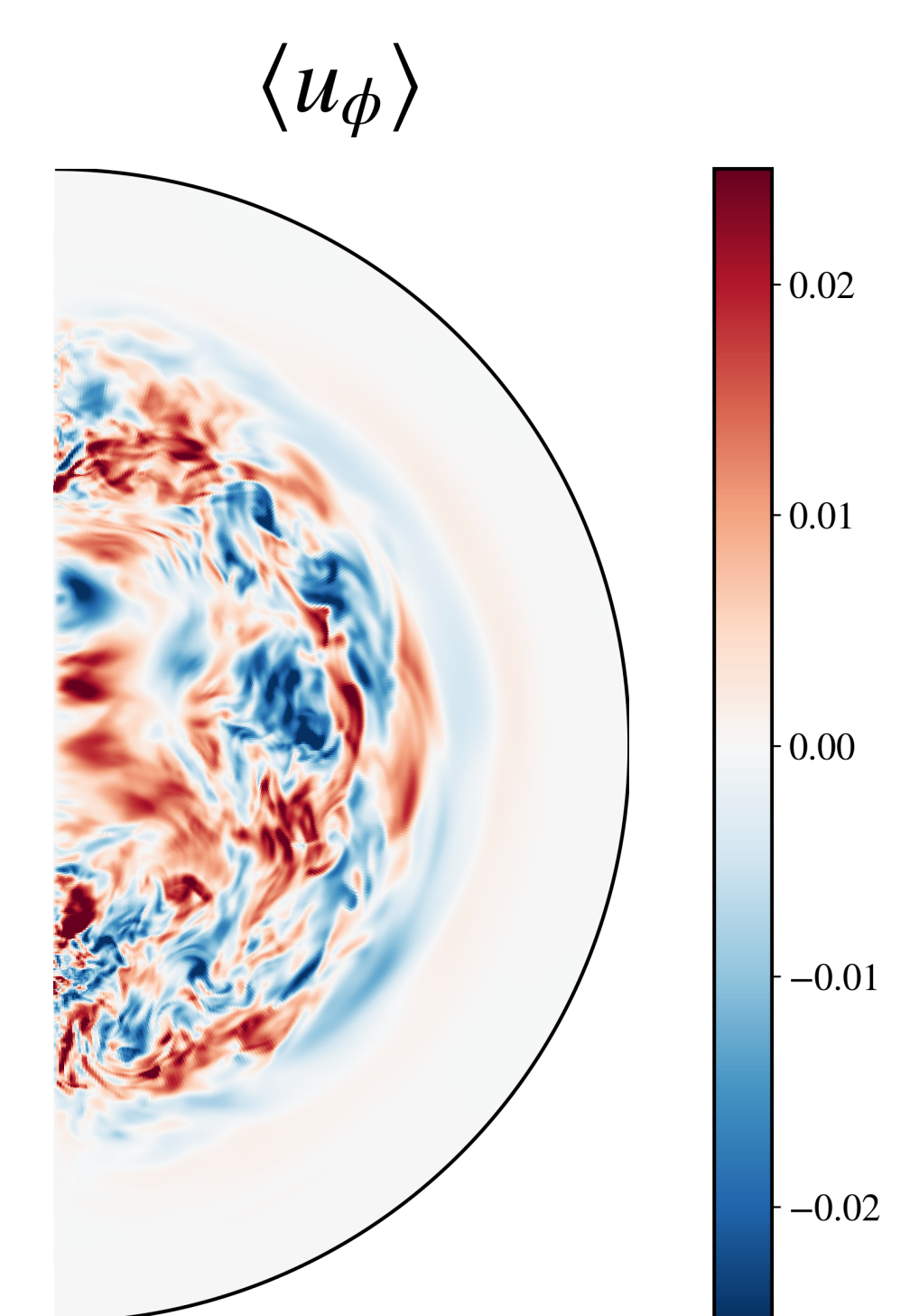


FIGURE 7 – Illustration of ongoing work where the current model (equatorial plane) is extended to the sphere to investigate the effect of rotation.

References

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