

Linear study of shear-driven instabilities in stellar radiative regions

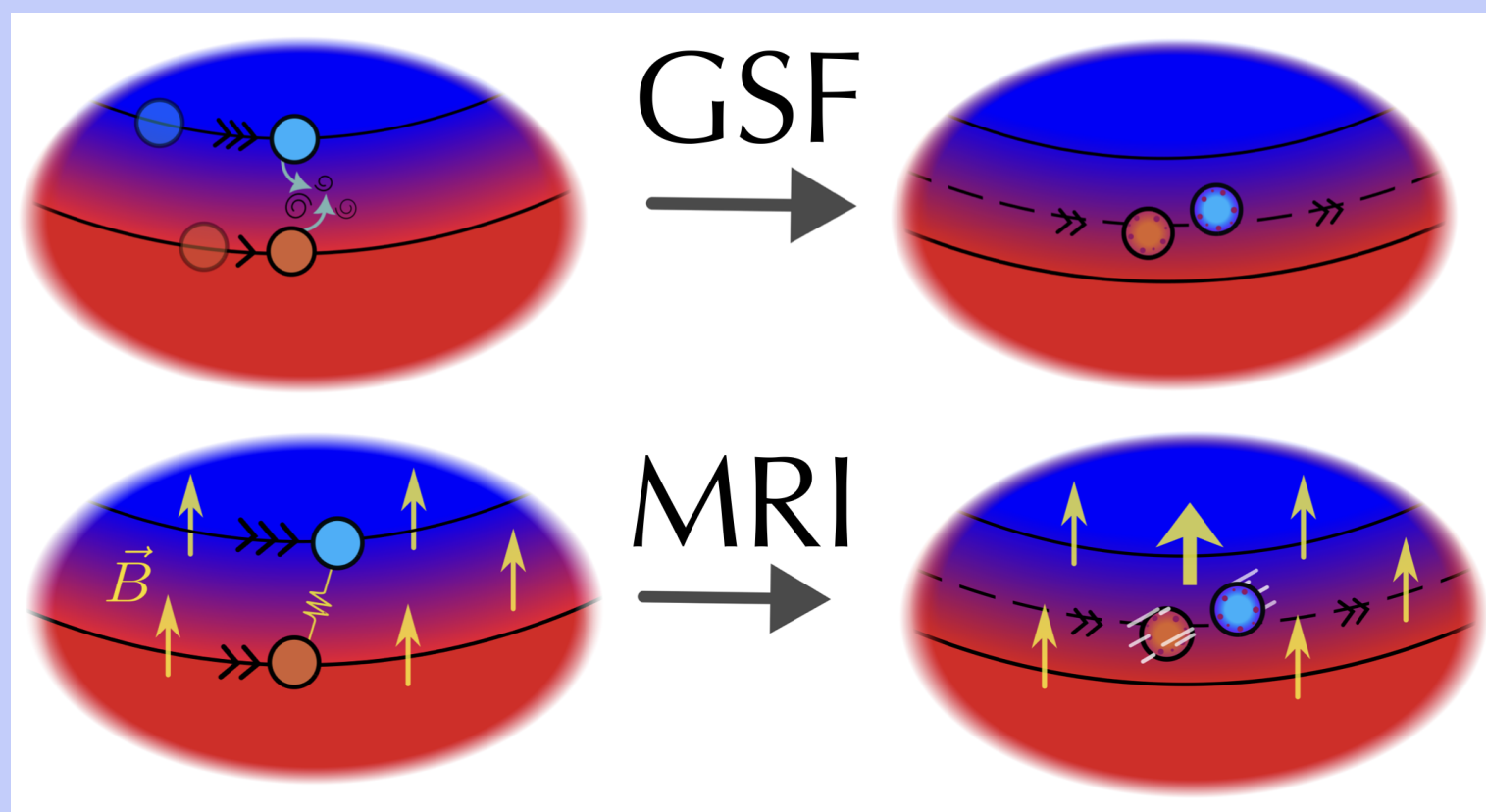
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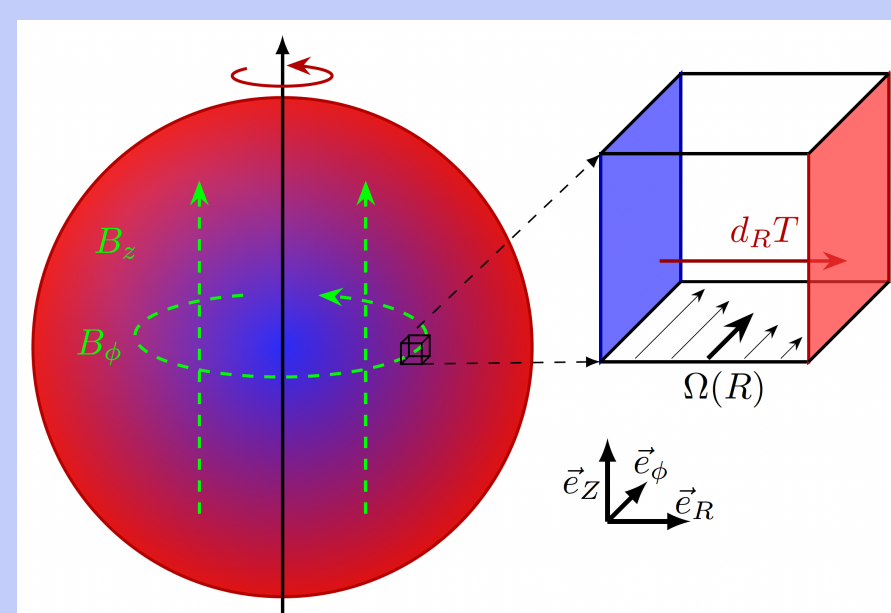
Introduction

Magnetohydrodynamic instabilities can play a key role in the transport of angular momentum and chemical species inside radiative regions of stars. They could also initiate a stellar dynamo explaining the origin of magnetism in radiative stellar layers. This poster focuses on shear-driven instabilities of the kind Goldreich-Schubert-Fricke (GSF) and Magneto-Rotational Instability (MRI).

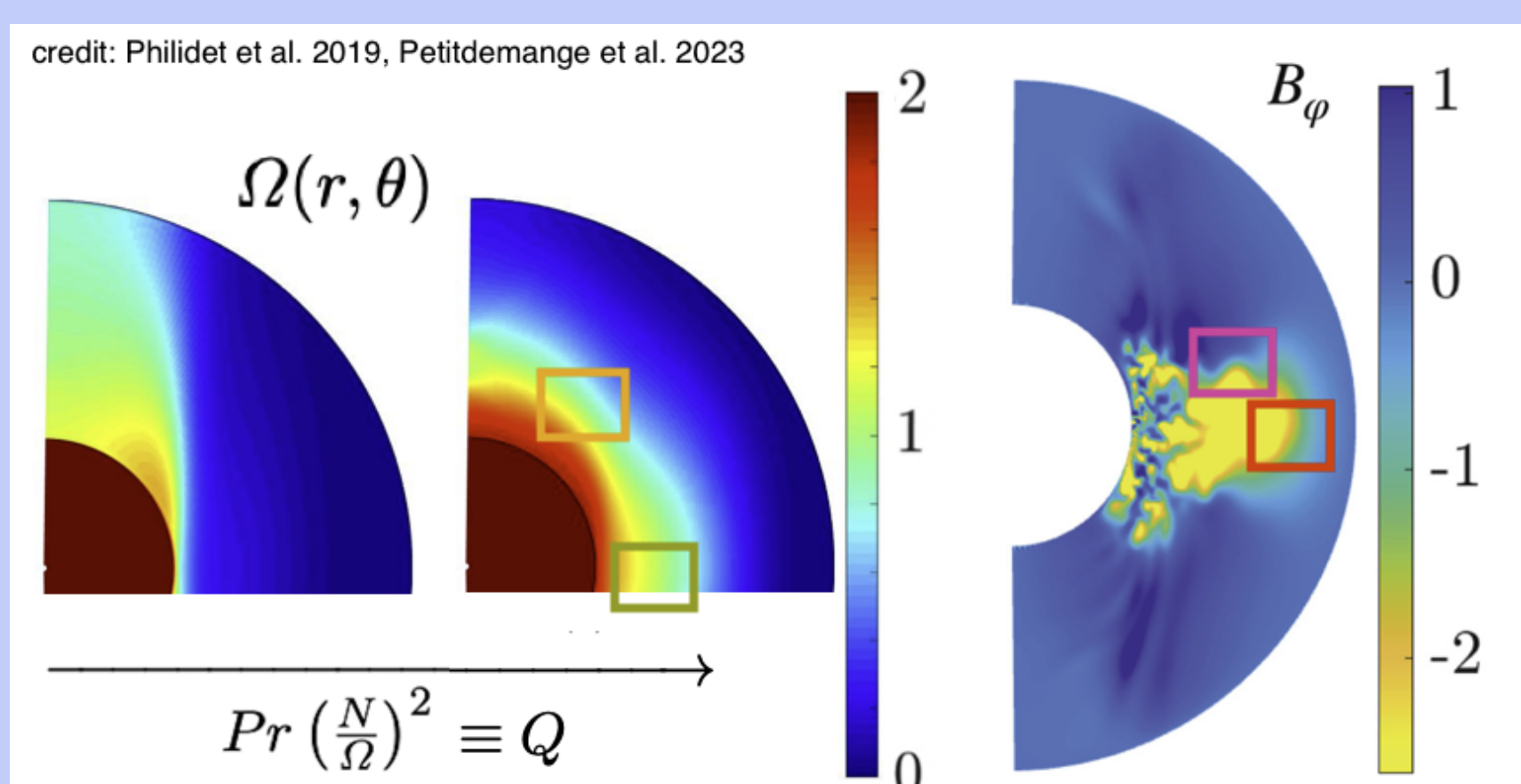


We aim to highlight the combined effect of stratification and magnetic tension on the development of these instabilities. Consider stellar regions with the following background state,

$U_0 = R\Omega(R)$, a magnetic field $B_0 = B_\phi(R) + B_Z$, and a temperature $T_0(R)$.



This approach focuses on equatorial regions¹, although other instabilities, such as the Taylor Instability, may also develop elsewhere.



MHD & Thermal equations

$$\partial_t U + U \cdot \nabla U = -\frac{1}{\rho} \nabla P + \frac{1}{\rho \mu} (\nabla \wedge B) \wedge B - \rho g + \nu \Delta U$$

$$\partial_t B + U \cdot \nabla B = B \cdot \nabla U + \eta \Delta B$$

$$\partial_t T + u \cdot \nabla T = \kappa \Delta T \quad \text{with } \rho(T) = \rho(1 - \alpha T)$$

We consider perturbations of the form,

$$A = A_0 + a' \quad \text{with } a' \propto a e^{i(k_R R + m\phi + k_Z Z) + \sigma t}$$

The resulting eigenvalue problem² is $\mathcal{H}v = \sigma v$, with $v = (u'_R, u'_\phi, b'_R, b'_\phi, \theta')$

Notations

Dimensionless parameters, together with their estimated values in radiative regions and typically used in simulations :

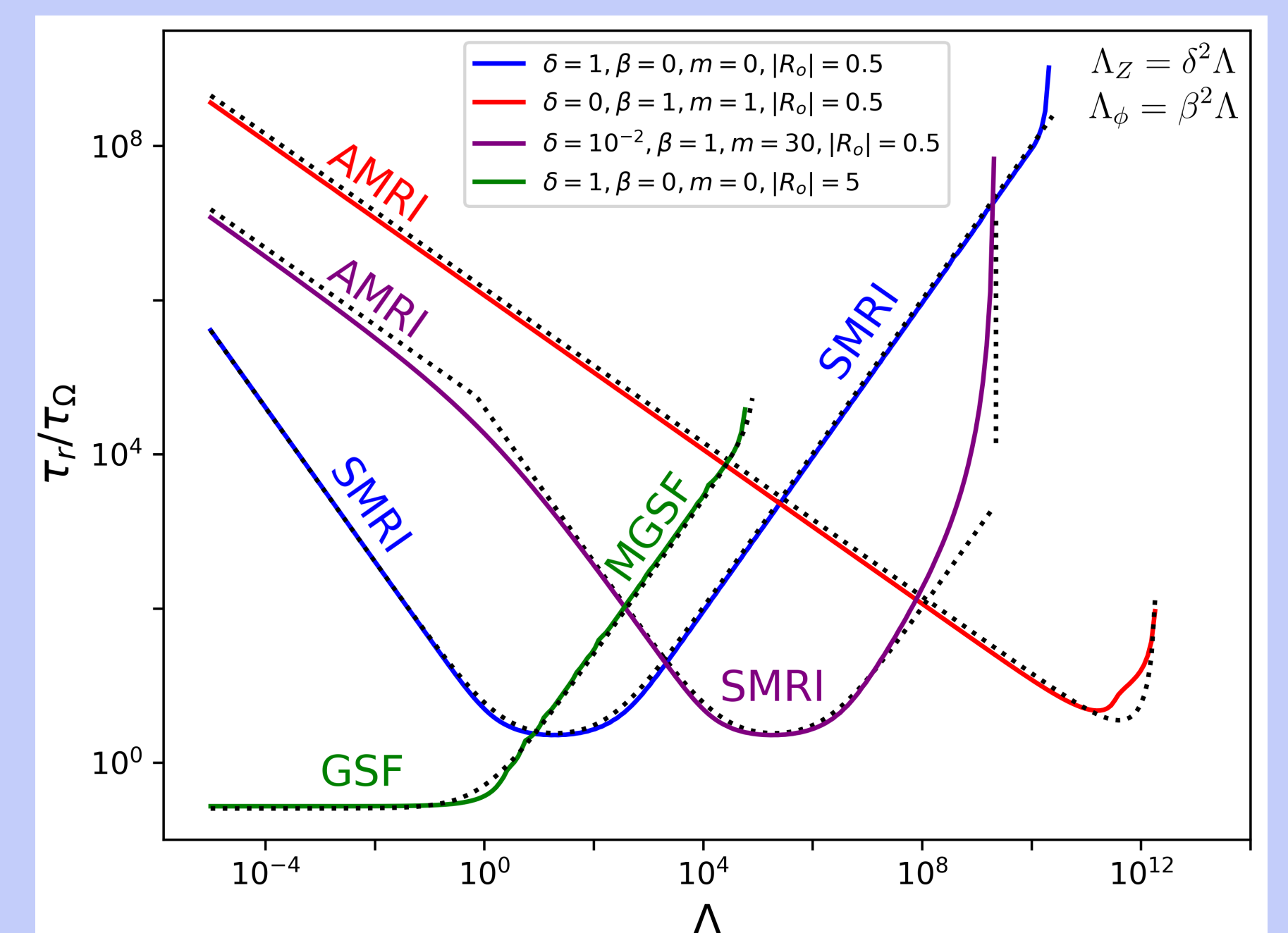
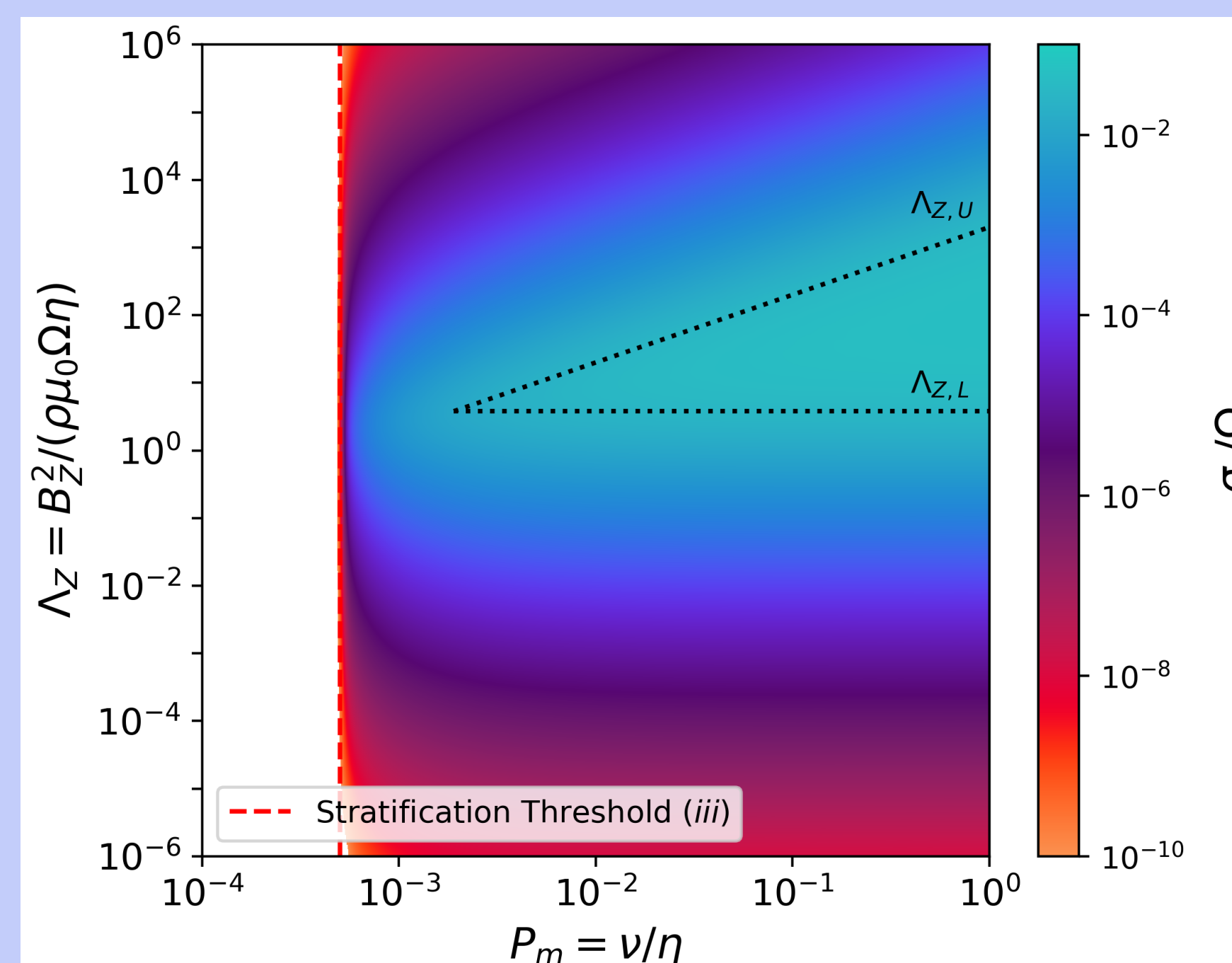
Parameter	Ratio	Rad.	Simu.
Prandtl	$P_r = \frac{\nu}{\kappa}$	10^{-5}	10^{-1}
Magnetic Prandtl	$P_m = \frac{\nu}{\eta}$	10^{-3}	1
Ekman	$E = \frac{\nu}{\Omega L^2}$	10^{-15}	10^{-5}
Elsässer	$\Lambda = \frac{B^2}{\mu \rho \Omega \eta}$	1	1
Stratification	$\frac{N^2}{\Omega^2} = \frac{\alpha g \partial T}{\Omega^2 \partial r}$	10^3	10

We note, $R_o = \frac{R}{2\Omega} \frac{\partial \Omega}{\partial R}$, $R_{b,R} = \frac{R^2}{2B_\phi} \frac{\partial B_\phi / R}{\partial R}$ and similarly for $R_{o,Z}$, $R_{b,Z}$

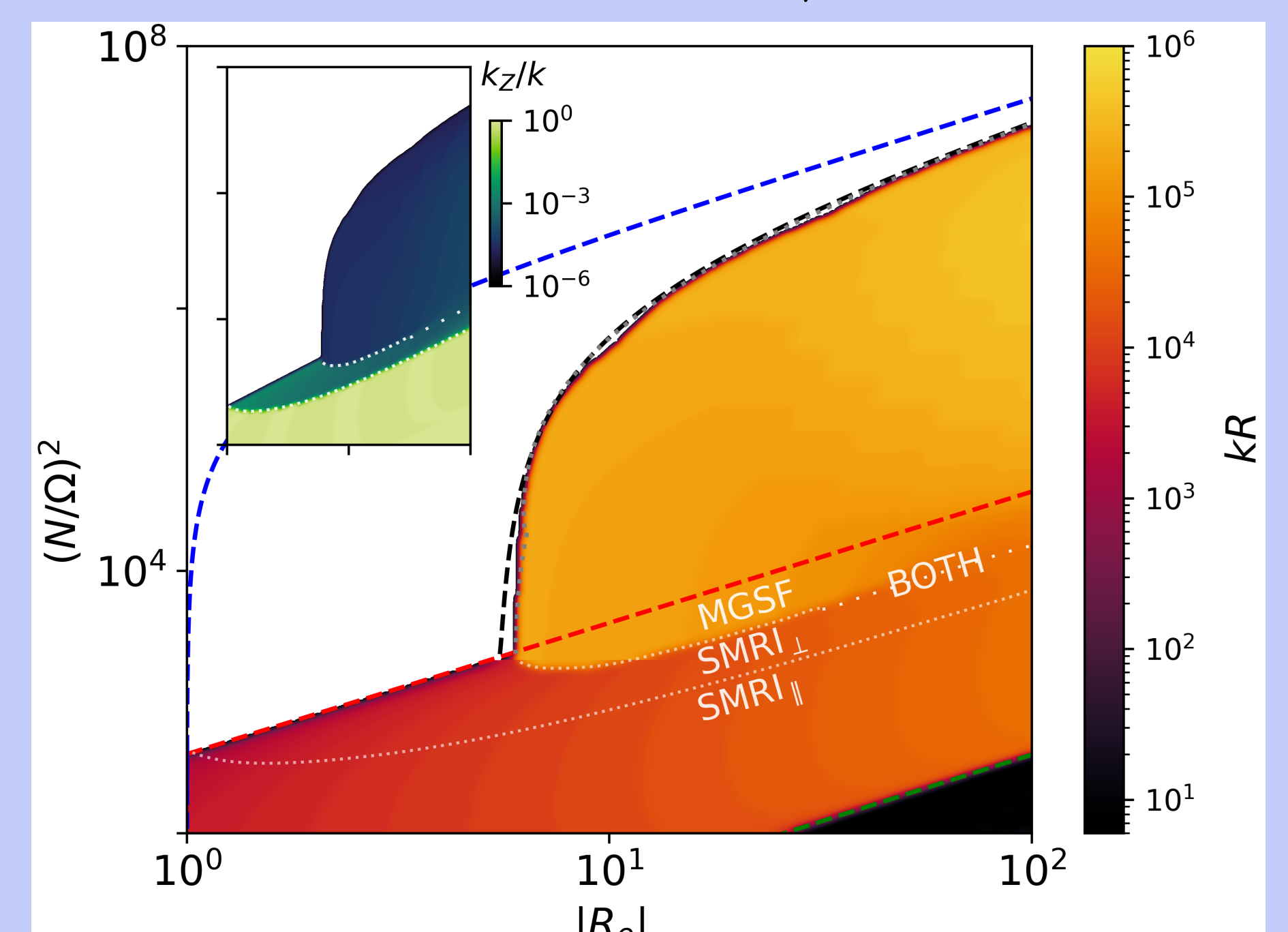
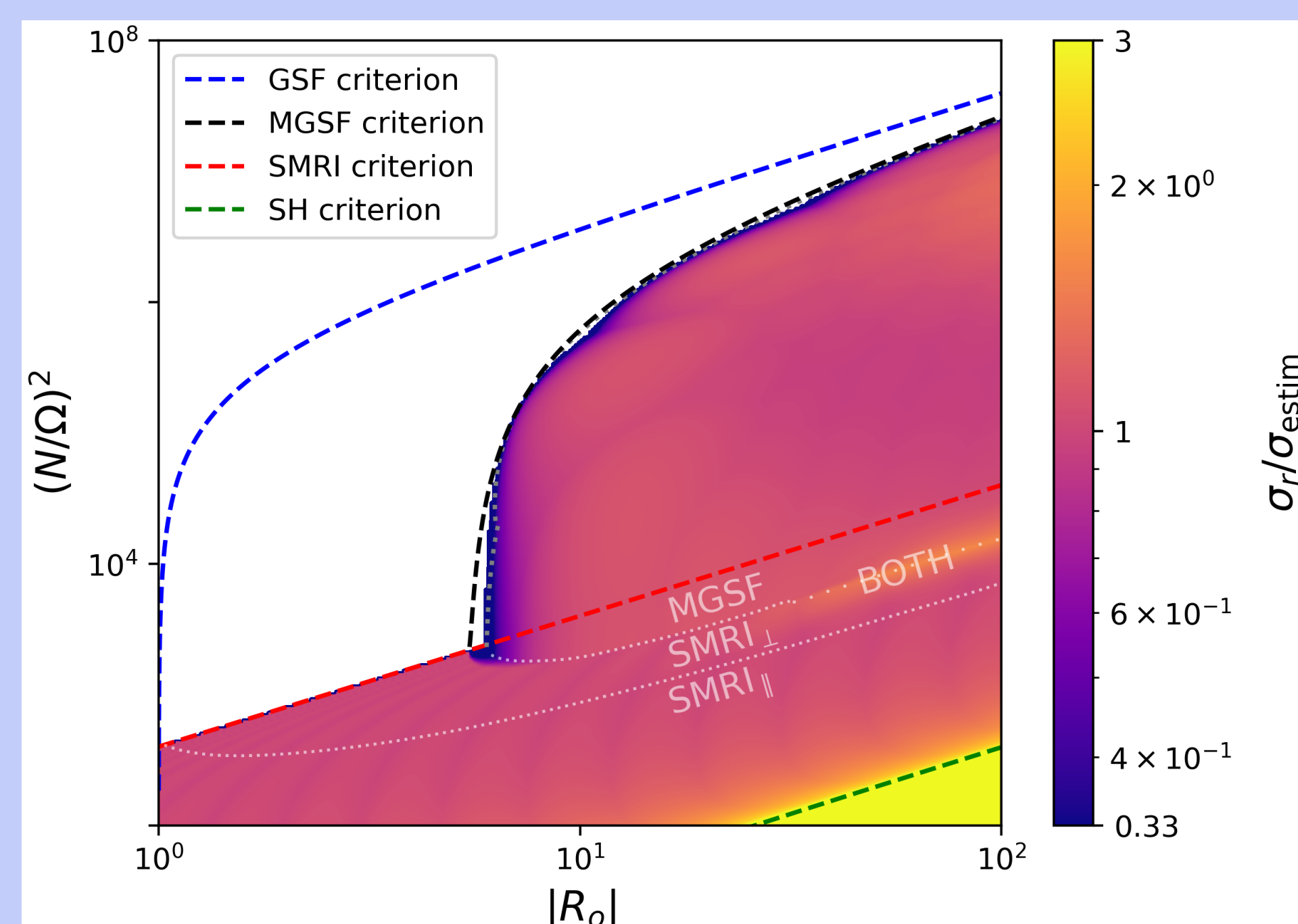
Linear approach of MRI and GSF instability

In stellar radiative regions, the main stabilizing mechanism is stratification¹, quantified by $(N/\Omega)^2$. In weakly sheared regions, we provide a more general picture of the MRI by identifying,

- the stabilization of the standard MRI ($B = B_Z$) in the magneto-buoyant regime³ $\Lambda > \Lambda_{Z,U} = 4P_m R_o / P_r (N/\Omega)^2$, where the growth rate scales as $\sigma_{SMRI} \propto \Lambda_{Z,U} / \Lambda$.
- a weaker effect on the azimuthal MRI ($B = B_\phi$) modes, for which growth rates depend less strongly on stratification.

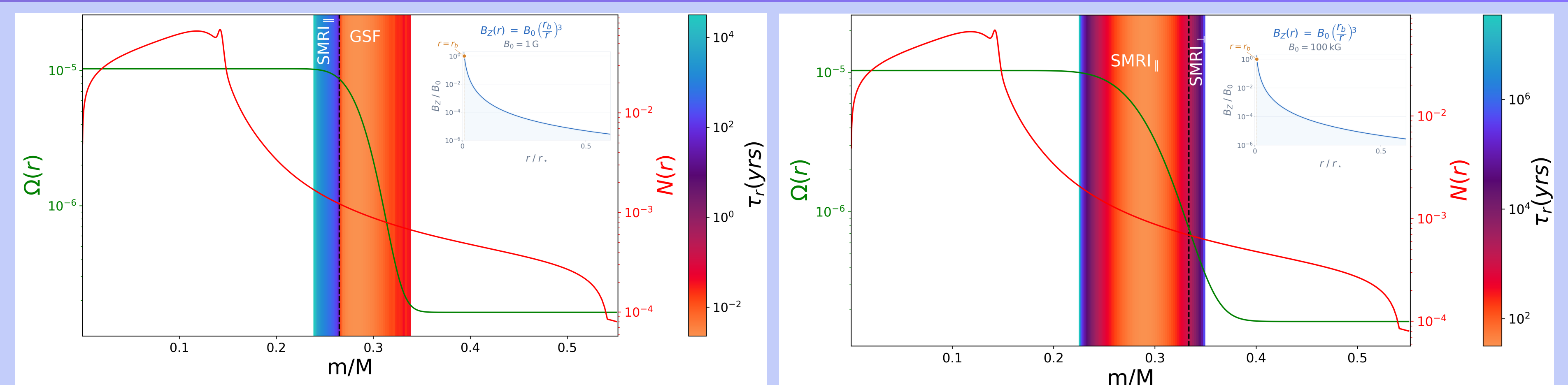


In strongly sheared regions, we identified a new stability criterion for the magnetized GSF instability associated with magnetic tension and magneto-buoyant effects that stabilize these modes⁴. In strongly magnetized regimes, the growth rate behaves as, $\sigma_{MGSF} \propto \sqrt{|R_o| - 1/\Lambda}$.



In parameter space, the transition between SMRI and MGFSF modes occurs through intermediate SMRI_⊥ modes, sharing growth properties with the SMRI, but with wavevectors oriented nearly orthogonally to B_Z , similarly to the MGFSF modes. We note $\sigma_{estim} = \max(\sigma_{SMRI}, \sigma_{MGSF})$.

Application to models of early red giant stars



With the stellar evolution code Cesam2k20, models were computed for a set of early red giant stars (constraint on their rotation profiles⁵ and magnetic fields⁶) and we estimated the development of SMRI and GSF modes. **Stratification and magnetic tension significantly stabilize the instabilities considered here, although they may still grow on short timescales for magnetic fields of order 100kG.** This suggests that these mechanisms may operate readily in such stars.

Conclusion & Outlook

This work improves our understanding of shear-driven instabilities in magneto-buoyant regimes relevant to stellar radiative regions³. We show that **stratification and magnetic tension strongly modify and stabilize MRI and GSF modes, although these instabilities may still operate efficiently in stellar interiors.** Future work will investigate their nonlinear saturation and their interaction with other MHD instabilities using 3D direct numerical simulations.

References

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