Numerical treatment of the Tayler-Spruit dynamo in stellar evolution codes

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Abstract

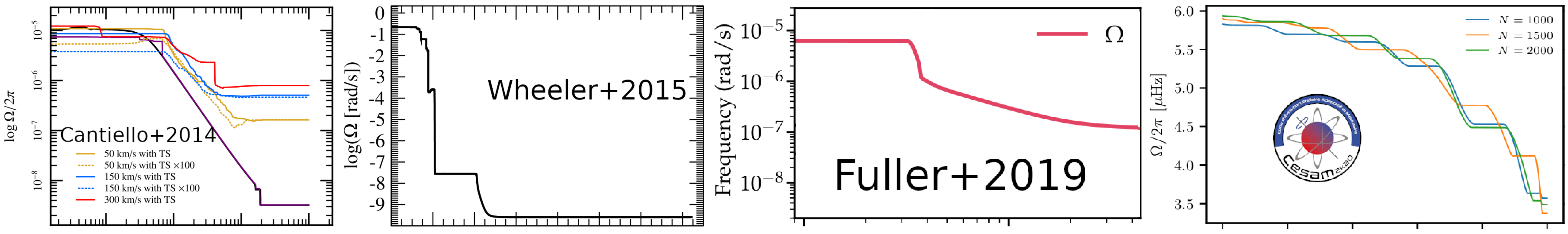
The transport of angular momentum in stellar radiative zones is traditionally modelled as the result of the meridional circulation advecting angular momentum, and shear-induced turbulence diffusing angular velocity (Zahn 1992):

$$\rho\frac{\mathrm{d}r^2\Omega}{\mathrm{d}t} = \frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4\Omega U_2\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho\nu r^4\frac{\partial\Omega}{\partial r}\right). \tag{1}$$

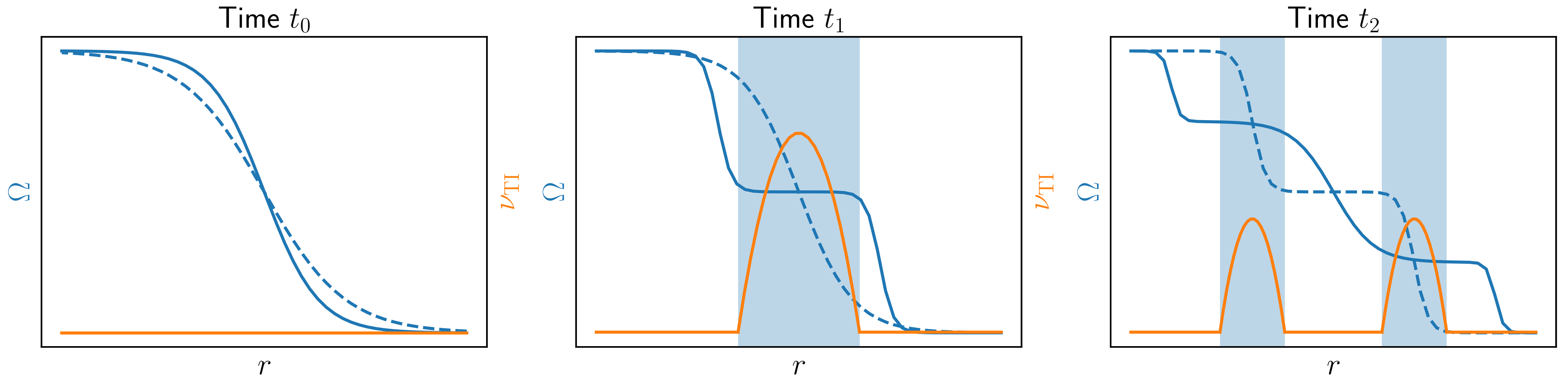
This model traditionally fails at reproducing rotation rates in observed stars, suggesting additional transport mechanisms must be considered. Many candidates exists, and among them the diffusion induced by the Tayler-Spruit (TS) instability.

Triggered when $q \equiv \frac{\partial\ln\Omega}{\partial\ln r} > \left(\frac{N}{\Omega}\right)^{7/4}\left(\frac{\eta}{r^2N}\right)^{1/4} = q_{\rm min},$ which induces a viscosity $\nu_{\rm TS} = \frac{\Omega r^2}{q}\left(\frac{\omega_{\rm A}}{\Omega}\right)^3\frac{\Omega}{N}.$

Stairway to Hell



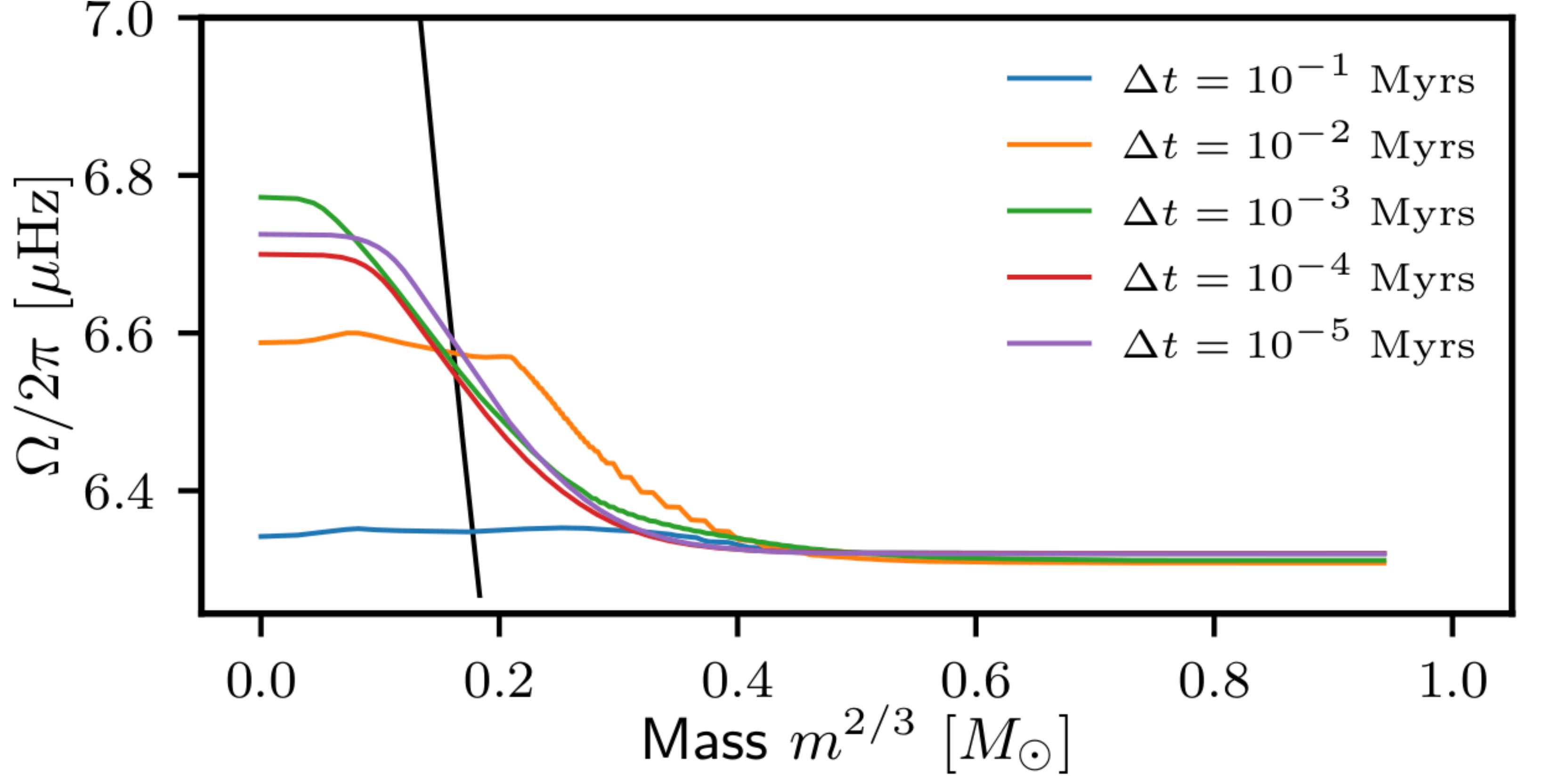
Ω-profiles look like stairs, purely numerical. Why ?



When solving Eq. (1), $\nu_{\rm TS}$ is computed using Ω at previous time for stability. Because of this time decoupling and because $\nu_{\rm TS}$ depends heavily on Ω and reaches high values ($\simeq 10^8$ cm²s⁻¹), evolution Ω should be computed with very short time steps ($\simeq 1-10$ yrs) compare to evolution time steps ($\simeq 10$ Myrs).

Short-timescale transport of Ω with CESTAT

CESTAT is a *Cesam2k20-derived STAtic code with Transport*. It solves Eq. (1) without evolving the stellar structure.



Starting from a model with Ω as the black line (linear in $m^{2/3}$), Eq. (1) is solved for a total of 1 Myrs, decomposed in time steps Δt of 10^{-1} to 10^{-6} Myrs. When Δt is sufficiently short, numerical artefact disappear.

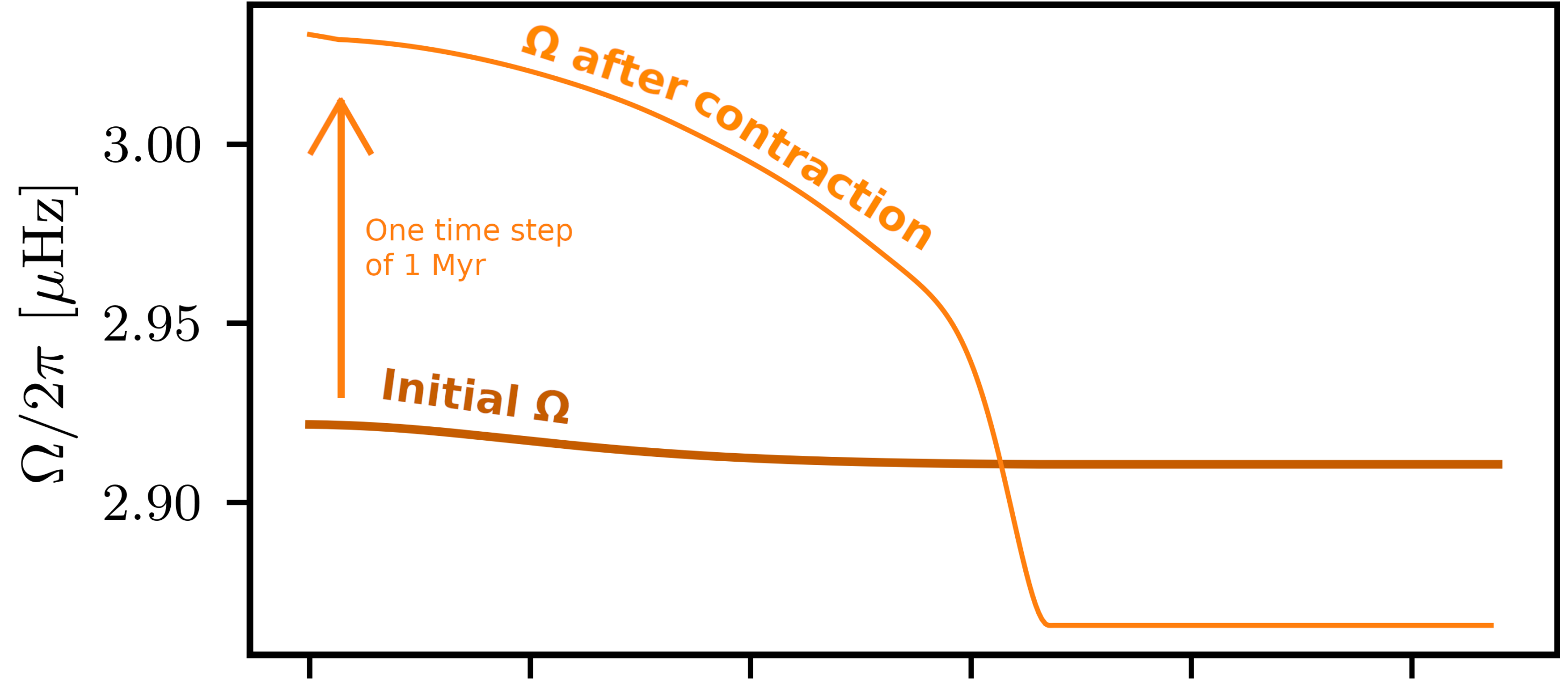
Perspectives

- Implement automatic double time-step method;
- Reassess efficiency of TS dynamo and other efficient processes.
- Study the induced transport of chemicals.

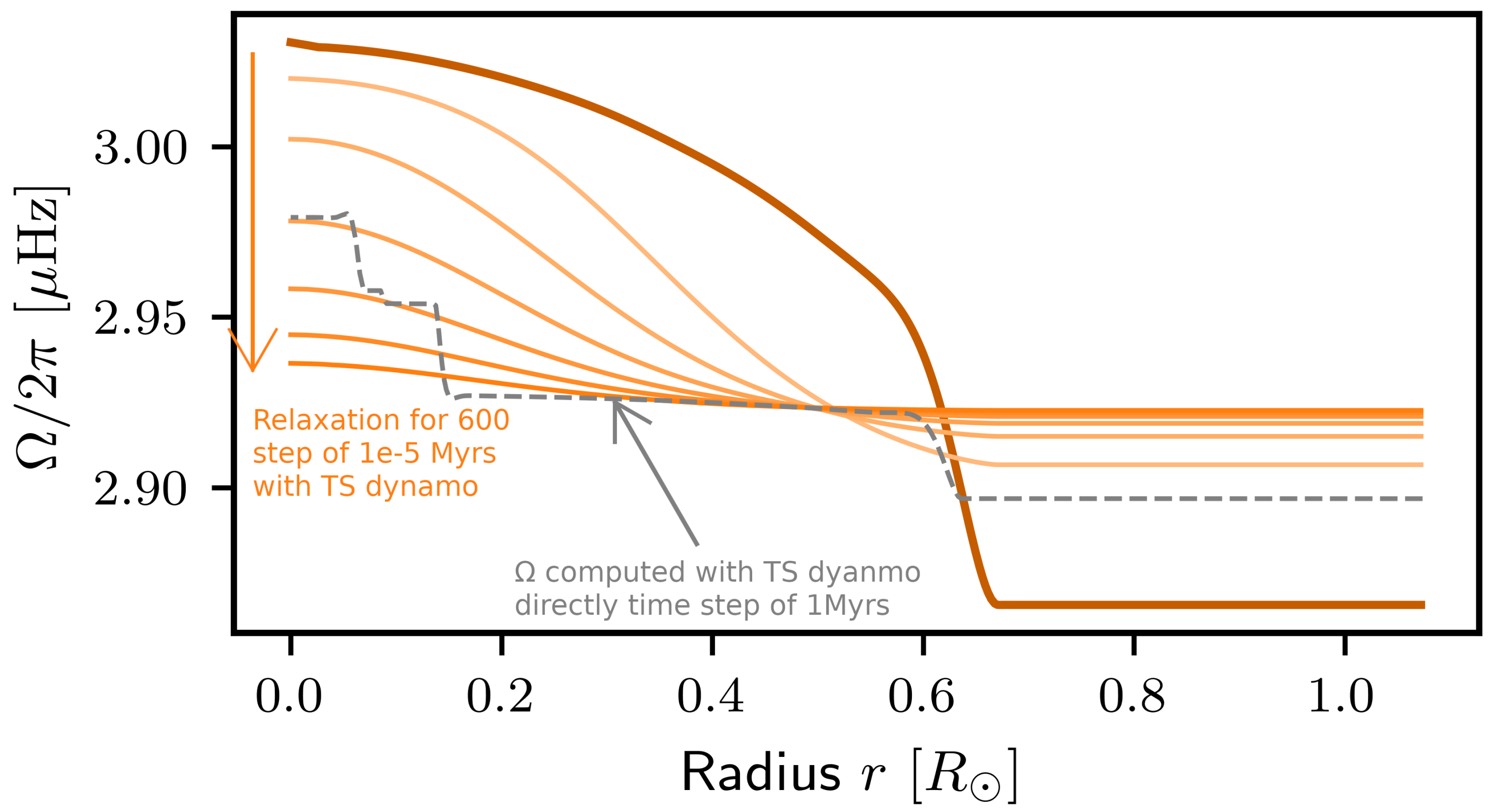
4th Cesam2k20 Workshop:

Double-timestep evolution

Evolution of the structure (ex. of a PMS model) with Cesam2k20 and solve Eq. (1) with RHS $= 0$.



Then solve Eq. (1) with CESTAT with time step of 10 yrs accounting for TS dynamo



⇒ A double-timestep method yield a smooth Ω where TS dynamo is found to be more efficient (during PMS) than when the timestep follows the evolution timescale.