Systematic study of the low mass stars' rotation profiles

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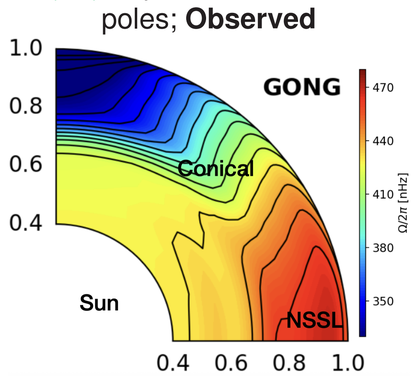
Aims

1. Unveiling the rotation profile during the transition from a (SL) to an (AS) rotation profile at the end of the low mass star Main Sequence (MS).
2. How the heat is it transported before, during and after the transition?

0. Introduction

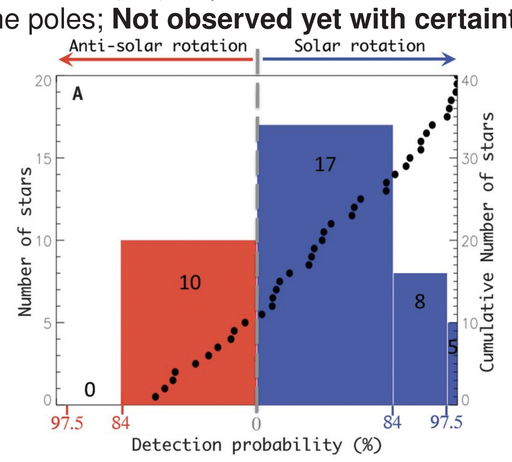
2 rotation profiles observed/expected for the low mass stars :

Solar-Like(SL): equator rotates faster than the poles; **Observed**

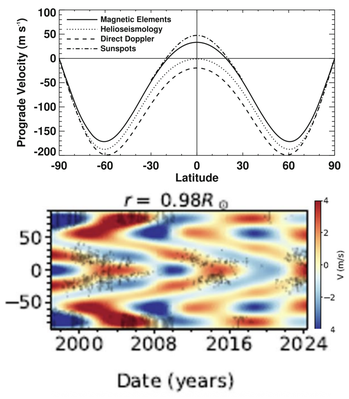


Sun's rotation profile adapted from [1]

Anti-Solar(AS): equator rotates slower than the poles; **Not observed yet with certainty**



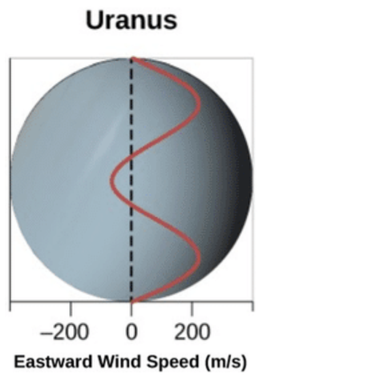
(SL) & (AS) detection probability [2]



Sun's surface rotation profile and Torsionnal Oscillations (TO) [4, 5]

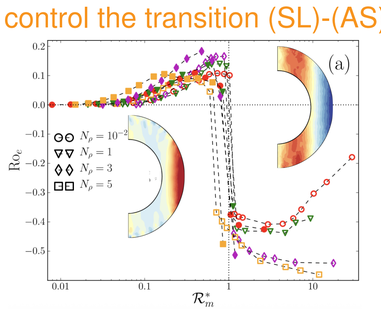
2 questions still in debate in stellar physics [1]:

- Reproduce the conical shape of the rotation at mid-latitude in the 3D numerical simulations
- Reproduce the Near Sub-surface Shear Layer (NSSL) in the 3D numerical simulations

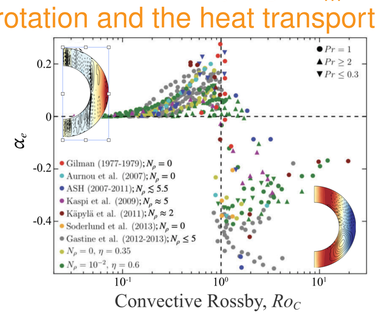


Uranus' rotation profile: [3]

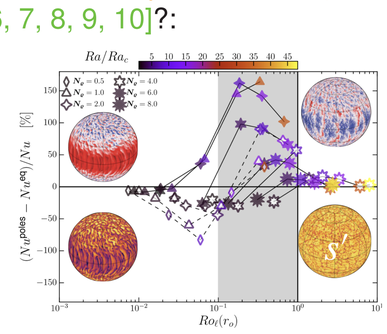
The convective Rossby number Ro_c, modified Rayleigh R_m^* and local Rossby numbers $Ro_\cdot$, control the transition (SL)-(AS) rotation and the heat transport [6, 7, 8, 9, 10]?:



Amplitude of the zonal flow at the equator's surface *vs* R_m^* : [6]



Amplitude of the zonal flow at the equator's surface *vs* Ro_c [6, 10]



Surface's heat flux distribution *vs* $Ro_\cdot$: [9]

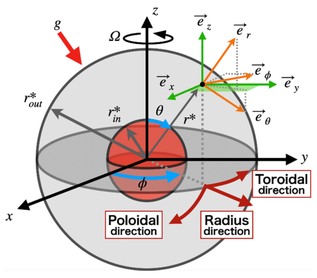
Limits:

- the parameter space is far removed from the stellar values: lower *Pr* and/or higher density contrast imposed N_ρ
- Ro_c does not explain the sharp transition for the transition (SL)-(AS) profiles
- the (NSSL) does not seem to appear in these simulations

I. Star considered, its numerical model [9] and equations [11]

Low mass star:= Radiative core (ignored) and convective envelope:

- Young red giant stars with a thick convective shell ($r \in [0.35, 1] r_o$)



Extracted from [13]

Navier-Stokes, mass, energy equations in the hydro. anelastic approximation:

- Navier-Stokes:

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{E_k}\nabla\left(\frac{P'}{\tilde{\rho}}\right) - 2\frac{1}{E_k}\Omega\wedge\mathbf{v} - \frac{\mathrm{Ra}}{\mathrm{Pr}}S'\mathbf{g} + \frac{1}{\tilde{\rho}}\nabla\cdot\mathrm{S}$$

- Where S is the Strain tensor
- Mass conservation:

$$\nabla\cdot(\tilde{\rho}\mathbf{v}) = 0$$

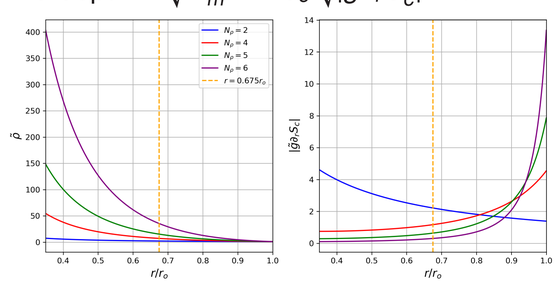
- Energy (Entropy form):

$$\frac{DS'}{Dt} = \frac{1}{Pr\tilde{\rho}\tilde{T}}\nabla\cdot\left(\tilde{\kappa}\tilde{\rho}\tilde{T}\nabla S'\right) + \frac{1}{\tilde{\rho}\tilde{T}}\frac{\mathrm{Pr}}{\mathrm{RaDi}}Q_\nu$$

- Where Q_ν is the viscous heating
- Adiabatic reference state

Characteristic values in the (CZ):

Parameter	Ratio	IRL.	Simu.
Prandtl	$Pr = \frac{\nu}{\kappa}$	10^{-5}	0.1
Rayleigh	$Ra = \frac{G\Delta S d^3}{c_P \nu\kappa}$	10^{30}	$10^5 - 10^7$
Ekman	$E_k = \frac{\nu}{\Omega d^2}$	10^{-15}	$3\cdot10^{-4}$
Density contrast	$N_\rho = \log(\tilde{\rho}_i/\tilde{\rho}_o)$	/	{2, 4, 5, 6}
Polytrop. index	n	/	2
Nusselt	$Nu = 1 + \frac{L_{conv}}{L_{rad}}$	/	/
Conv. Rossby	$Ro_c = \sqrt{\frac{RaE^2}{Pr}}$	/	/
Ra mid-depth	$\sqrt{R_m^*} = Ro_c\sqrt{\lvert g\partial_r S_c'\rvert}$	/	/



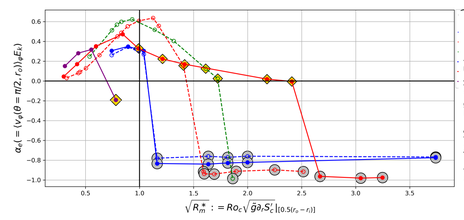
Adiabatic backgrounds for the density, the gravity and the conductive entropy (gradient)

Angular momentum equation:

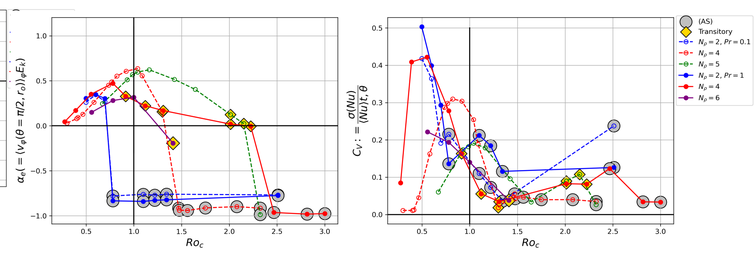
$$\tilde{\rho}\frac{\partial\langle\mathcal{L}\rangle}{\partial t} = -\nabla\cdot(\mathcal{F}_{Mer}) + \nabla\cdot(\mathcal{F}_{Re} + \mathcal{F}_\nu) \qquad (1)$$

$$\tilde{\rho}\langle\mathbf{v}_m\rangle_\varphi\cdot\nabla\mathcal{L} \simeq -\nabla\cdot(\tilde{\rho}r\sin(\theta)\langle\mathbf{u}'\cdot\mathbf{u}'_m\rangle_\varphi) \qquad (2)$$

II. Towards stellar parameters show some limits of the previous studies



Amplitude of the zonal flow at the equator's surface *vs* $\sqrt{R_m^*}$



Amplitude of v_φ at the equator's surface & the surface's heat flux *vs* Ro_c

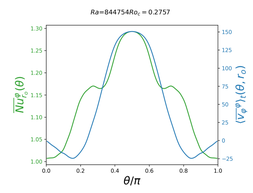
- Reproduction of the 2 rotation profiles as in the literature: a (SL) rotation profile for $Ro_c < 1$ and an (AS) one for $Ro_c > 1$ ... But
- The *Pr* (intrinsic properties of the fluid) and the density contrast delay the transition from a (SL) to an (AS) rotation profile even at $Pr = 1 \Rightarrow$ The Ro_c and R_m^* which are independent of the diffusivities do not catch properly the transition.
- The coefficient of variation of the axisymmetric heat flux $C_v = \sigma(Nu)/\langle Nu\rangle_{t,\varphi}$ shows the same trend when the $Ro_c \nearrow$
- **The simulations with a high N_ρ at $Ro_c \simeq 1$ are in transition and present 2 new rotation profiles**

3 different regimes for the heat flux distribution at the surface: Uniform (U), Poles hotter than the equator (PH), Equator hotter than the poles (EH):

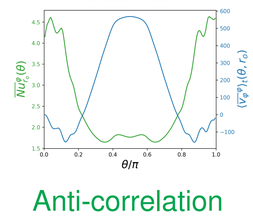
$$\text{(U) /(EH)} \xrightarrow[Ra>Ra_c]{Ro_c\nearrow} \text{(PH)} \xrightarrow[Ro_c\sim1]{Ro_c\nearrow} \text{(U)} \xrightarrow[Ro_c>1]{Ro_c\nearrow} \text{(EH)} \xrightarrow[Ro_c>>1]{Ro_c\nearrow} \text{(U)} \xrightarrow{Ro_c\nearrow} \text{(PH)}$$

The 2 last regimes are reached only at $N_\rho = 2$ due to the high level of turbulences.

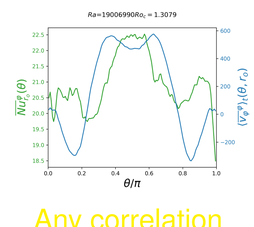
Examples of the results for $N_\rho = 4$, $Pr = 1$:



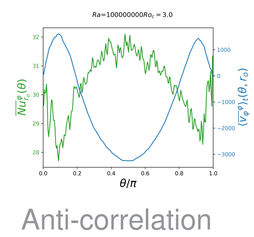
Correlation between *Nu* and v_φ ; $Ra \sim Ra_c$



Anti-correlation between *Nu* and v_φ , Coriolis regime: $Ro_c < 1$



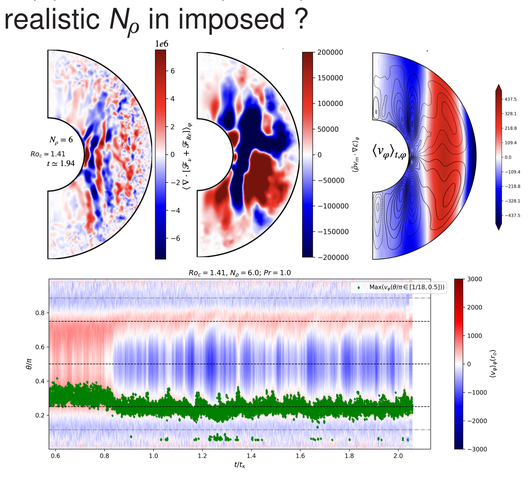
Any correlation between *Nu* and v_φ , Coriolis -buoyancy regime: $Ro_c \simeq 1$



Anti-correlation between *Nu* and v_φ , Buoyancy regime: $Ro_c >> 1$

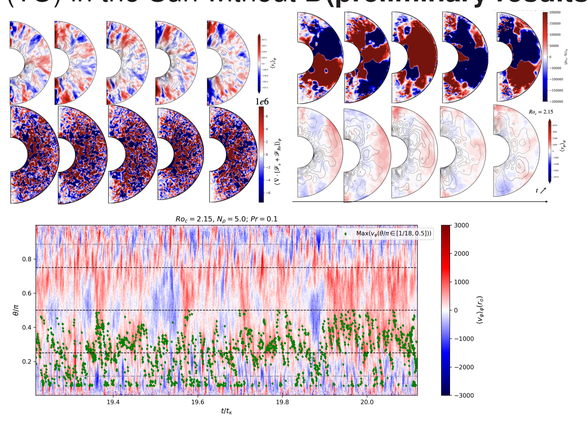
III. 2 new rotation profiles at $Ro_c \simeq 1$ and 1 new convective regime

I. (a) Toward the (NSSL) when a more realistic N_ρ in imposed ?



- 2 regimes coexist simultaneously: Coriolis within the tangent cylinder and the buoyancy outside the (TC)
- the system is balanced between the meridional circulation and the Reynolds stress at mid-depth in a steady state

I. (b) A cyclic inversion of the rotation profile between a (SL) and an (AS) regime like the (TO) in the Sun without $\vec{B}$ **(preliminary results)**

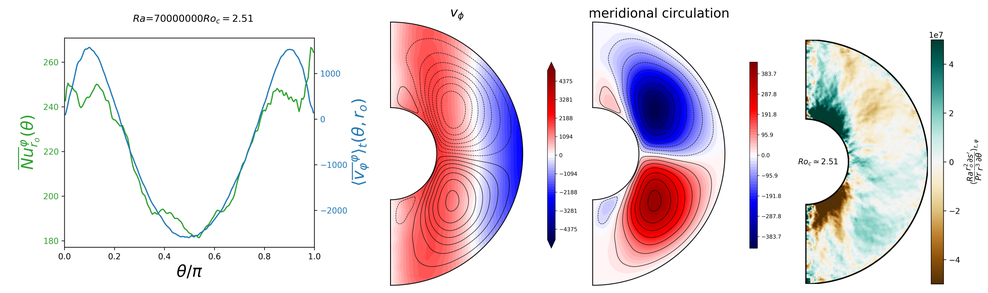


- the system is balanced between the meridional circulation and the Reynolds stress
- The inversion is due to lack of vigor of the imposed buoyancy (the *Ra*, so the v_r) which puts a stop to the meridional circulation to install a constant (AS) profile

Thermal wind equation :

$$\frac{\partial\langle u_\varphi\rangle_\varphi}{\partial z} \simeq \frac{Ra}{Pr}\frac{r_o^2}{r^3}\frac{\partial\langle s'\rangle_\varphi}{\partial\theta} \qquad (3)$$

II. A new convective regime: a correlation between the *Nu* and v_φ in an (AS) regime for the highest Ro_c



- Reason: A single large scale vortex is located along the rotation axis [14]

Conclusions / Perspectives

1. We have seen the limits of the Ro_c for the transition (SL)-(AS), and we want to improve this prescription by finding an other parameter and explain he 2 new rotation regimes.
2. By adding a radiative zone (RZ) below the (CZ) in order to be closer to the reality, we want to see if we will observe the conical shape of the rotation at mid-latitude in the (SL) and (AS) low mass stars as in [12]

References

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