Transport of angular momentum and chemical elements by magnetic fields in stellar radiative zones

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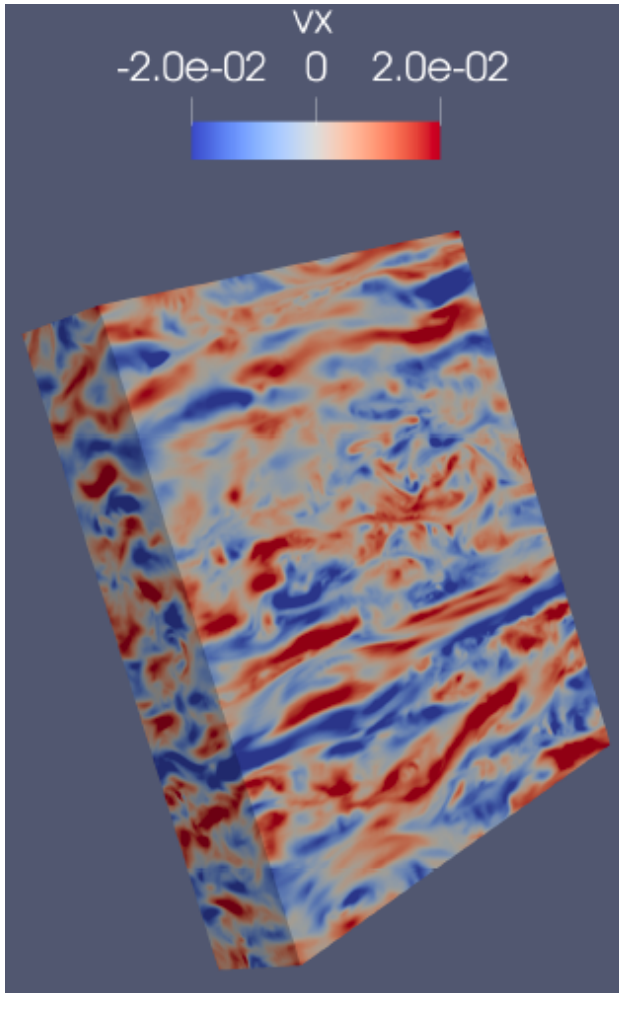

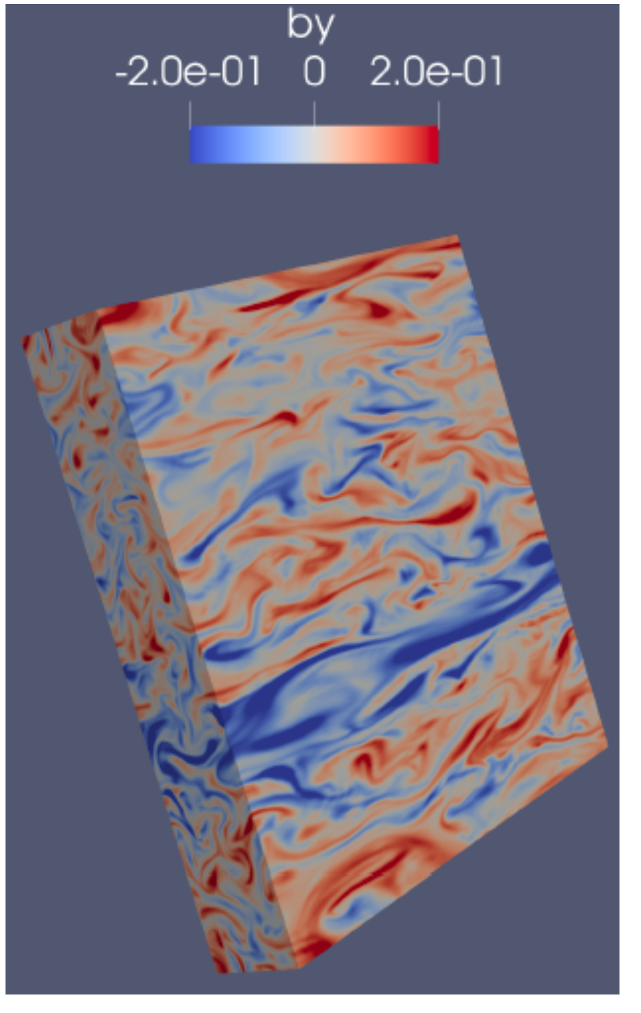

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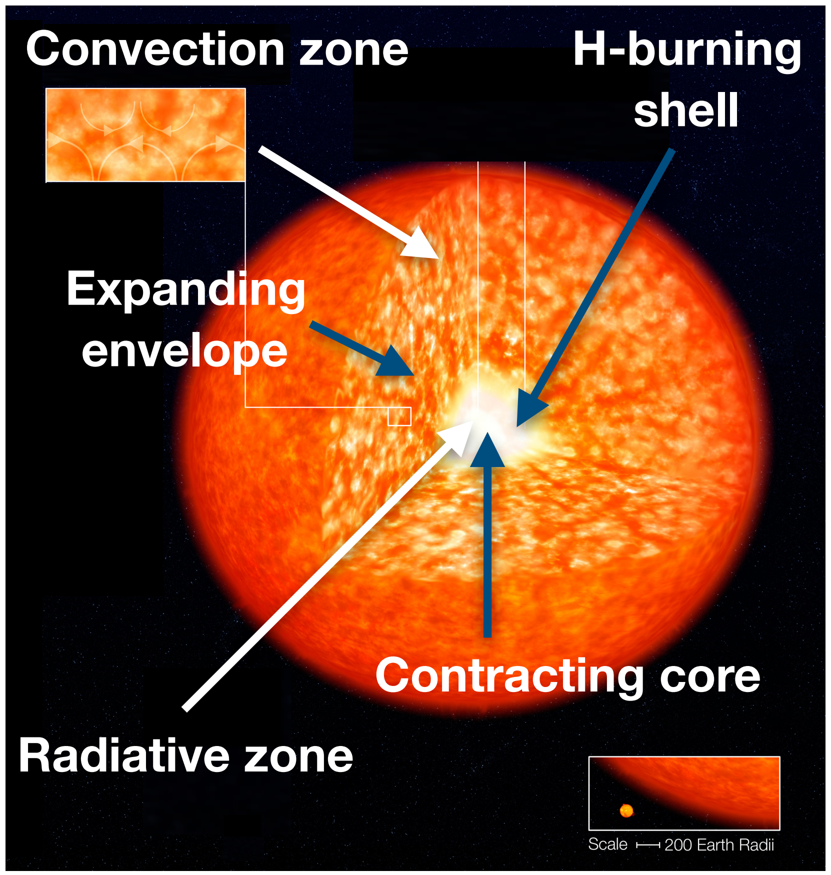
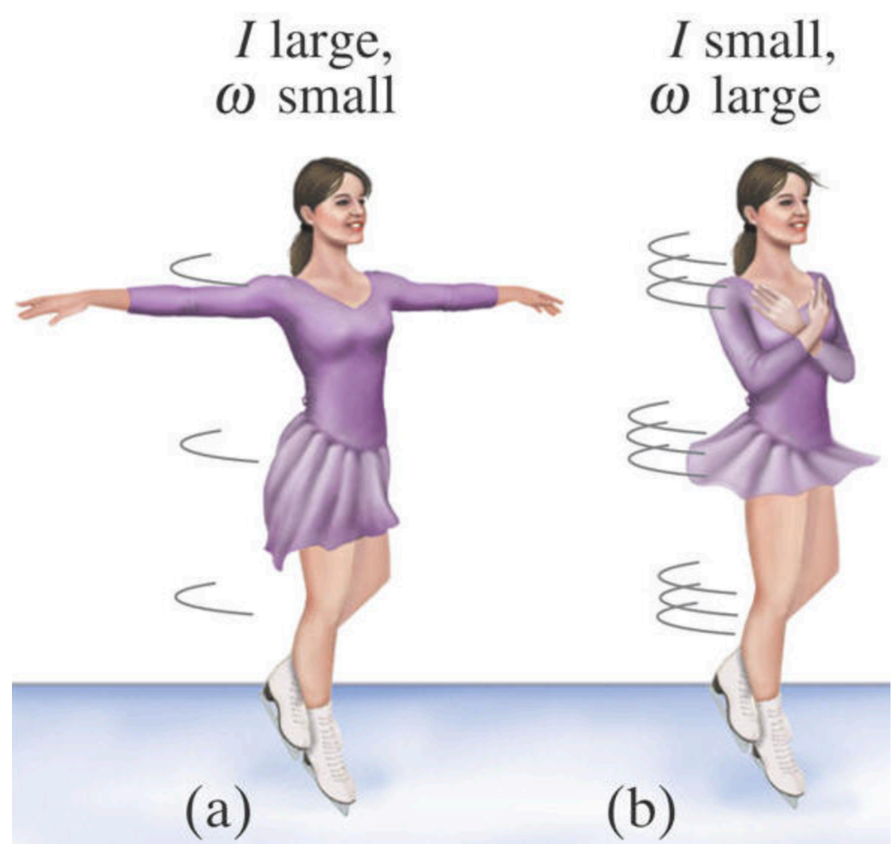
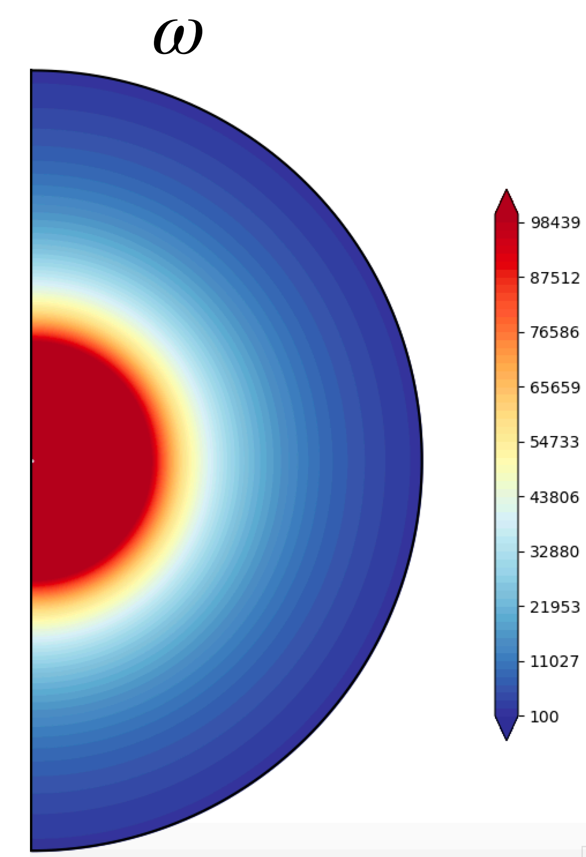
Astrophysical context of red giant stars: their core rotates much slower than predicted

✓ Core contraction => spin-up







✓ But…

- For young sub-giants: evidence for quasi-solid body rotation (Deheuvels et al. 2020)
- For more evolved sub-giants: Core spin-up and envelope spin-down with $\Omega_c \approx 5 - 15\ \Omega_{env}$ (e.g. Deheuvels et al. 2014)
- For red giants: Ω_c remains constant despite contraction (Gehan et al. 2018)

=> An efficient process at play to redistribute angular momentum at least during young sub-giant and red giant phases

Magnetism of red giant stars

✓ Recent discovery of fields of 100kG and more in the core of red giants

Li et al. 2022, Nature, Deheuvels et al. 2023, Li et al. 2023

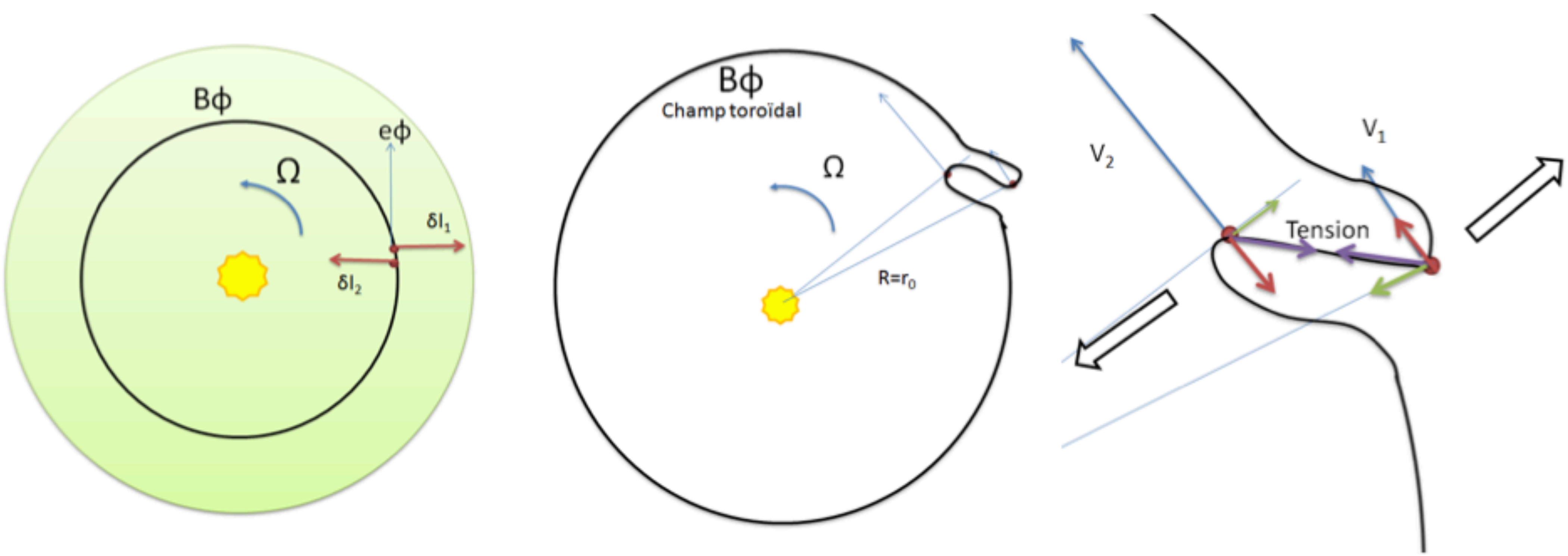
Field strengths from ~30 to ~600 kG with various geometries (not necessarily dipolar)

✓ MHD transport processes can thus be at play in red giants:

Alfvén waves, magnetic tension, MHD instabilities like the MRI:

MRI: Magneto-rotational instability (Velikhov 1959, Balbus & Hawley 1991)

- source of energy: kinetic energy of differential rotation (decreasing outward, AM increases outward)
- stabilised by strong magnetic fields



What do numerical simulations tell us?

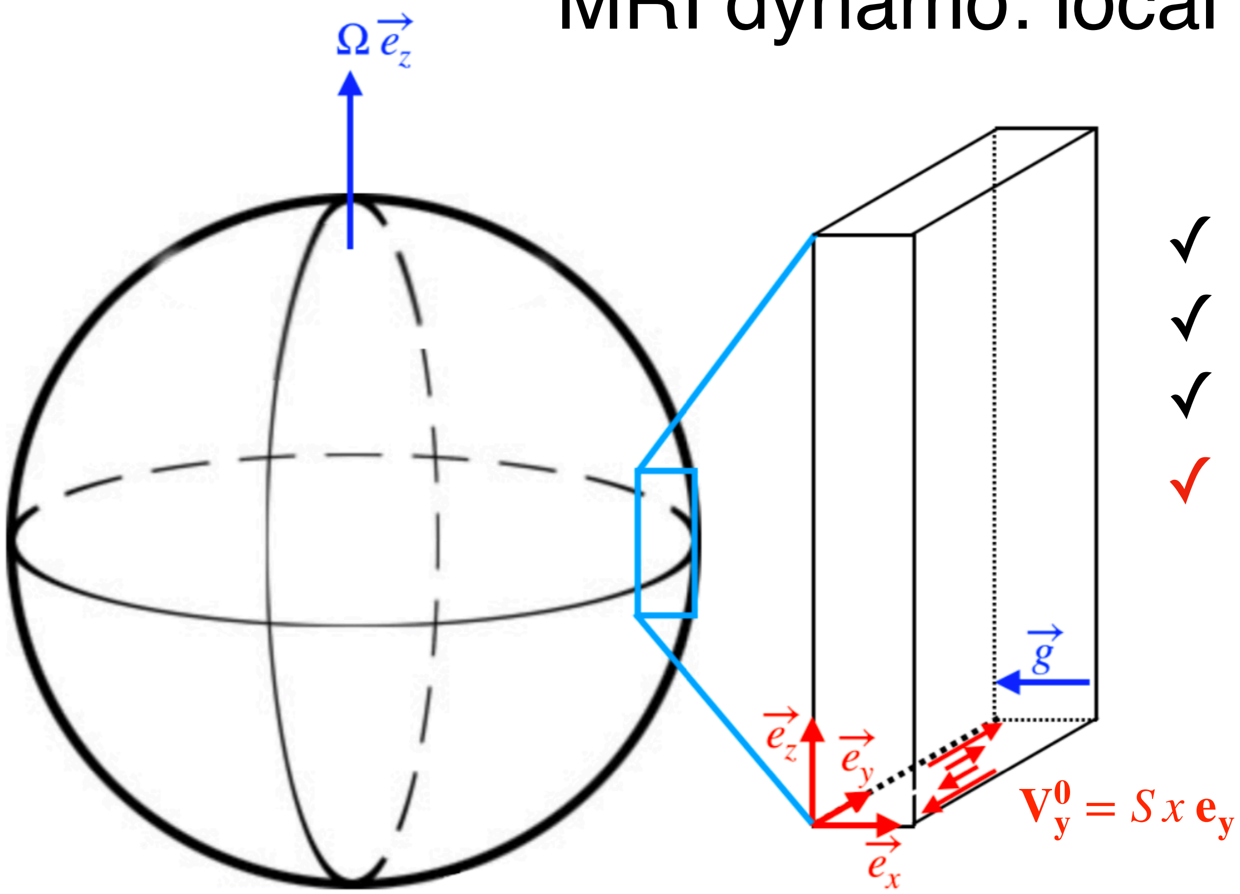
✓ MHD simulations with main ingredients:
- Differential rotation
- Initial magnetic fields
- Stable stratification (buoyancy as restoring force)
- Viscous, Ohmic dissipations and thermal diffusion terms

✓ 3 types of simulations:

- Forcing of differential rotation by contraction + initial large-scale field in sphere
  Gouhier et al. 2021, 2022
- Body-forcing of differential rotation + initial large-scale field in sphere
  Meduri et al. 2024
  Public code MagIC (pseudo-spectral 3D MHD, global spherical geometry)
  Wicht 2002, Gastine & Wicht 2012

- Body-forcing of differential rotation + initial noise (no net flux) in local boxes
  Jouve et al., A&A, under review
  Snoopy code (pseudo-spectral 3D MHD, local cartesian geometry)
  Lesur & Longaretti 2005

MRI dynamo: local shearing box simulations

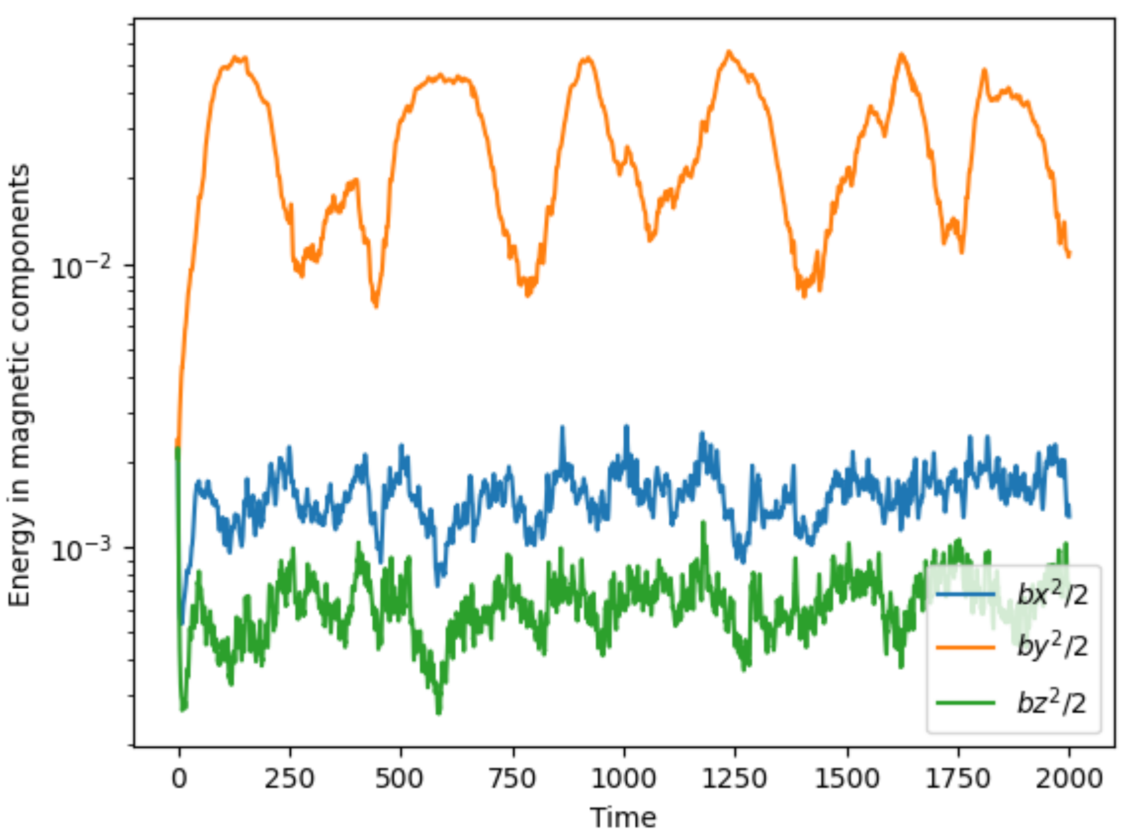


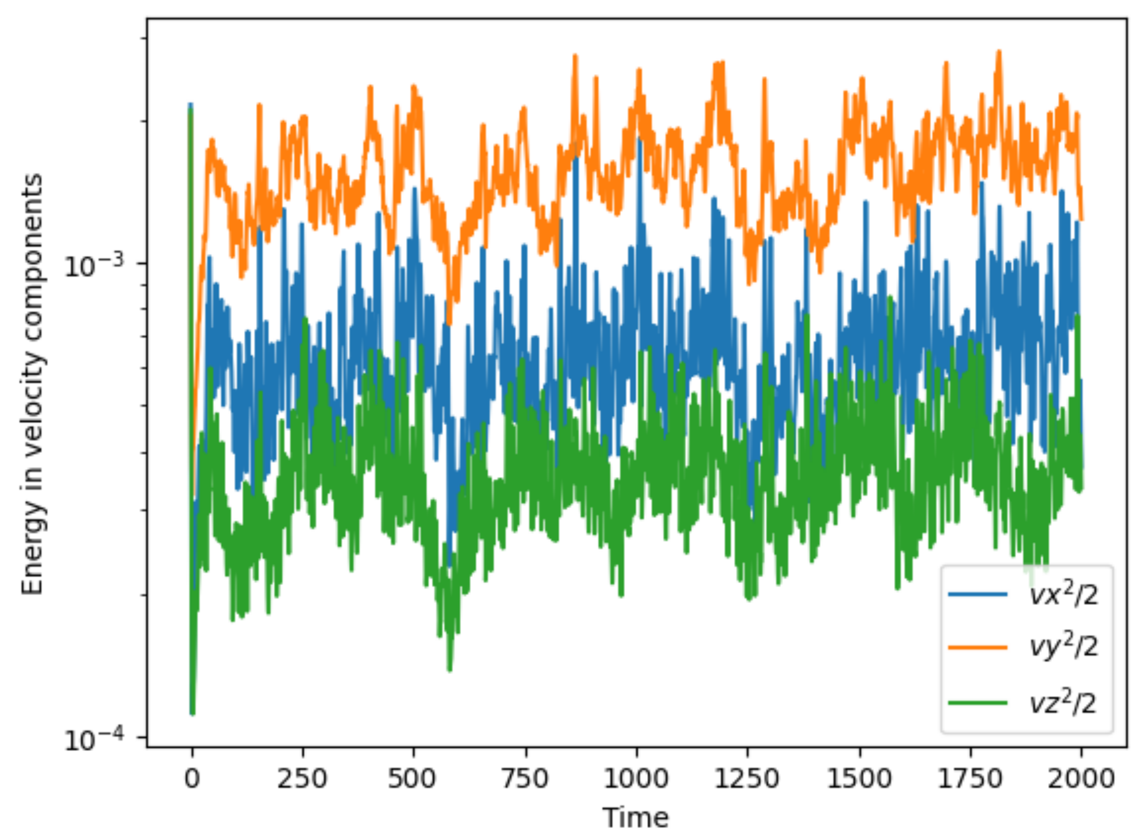
- ✓ Imposed shear $\mathbf{V_y^0} = S\,x\,\mathbf{e_y}$ where $S = -\,r\partial\Omega/\partial r$
- ✓ Initial conditions: white noise on flow and field
- ✓ Large Pm number to get dynamo action
- ✓ Test of impact of rotation Ω (z-direction)and stratification N (x-direction)

Snoopy
Lesur & Longaretti 2005

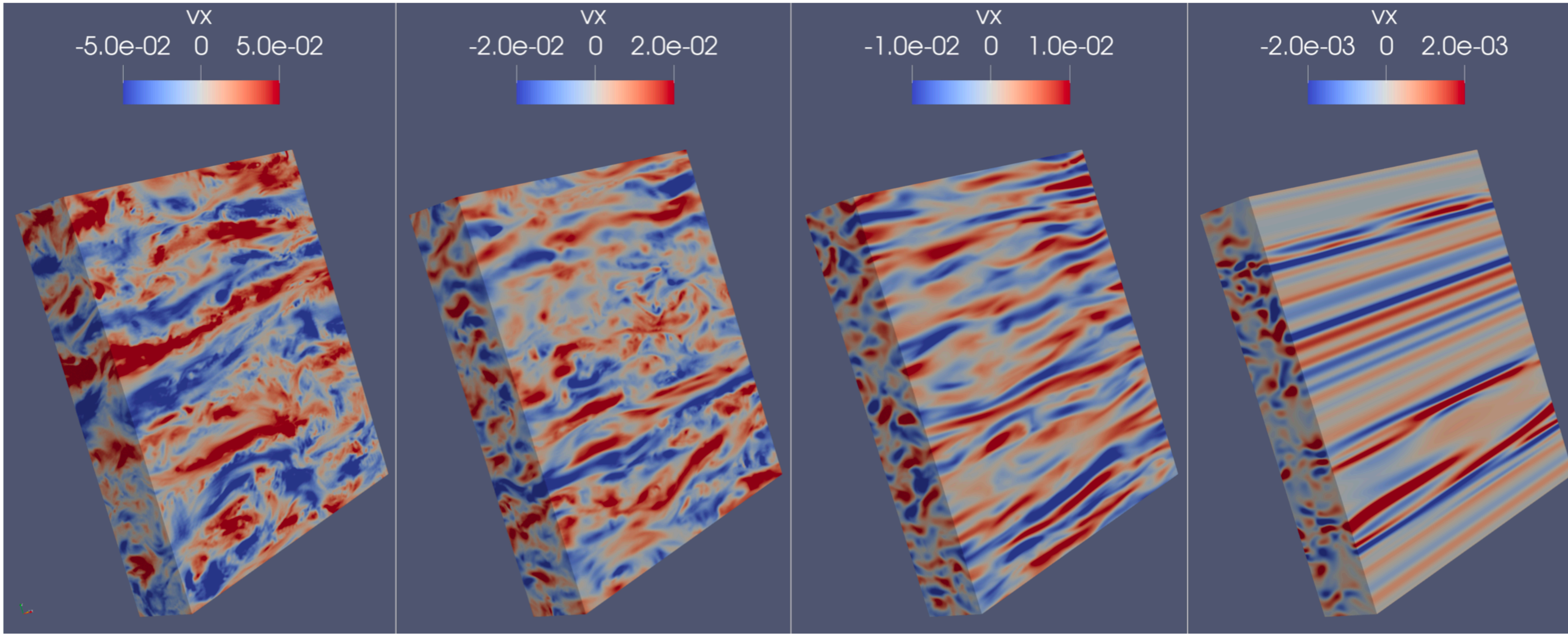
✓ Typical case at $Ro = S/\Omega = 1.5$ and $PrN^2/\Omega^2 = 0.028$: dynamo action!

Compatible with previous unstratified simulations (e.g. Rincon et al. 2007, Lesur & Ogilvie 2008)





Impact of stratification: amplitudes and lengthscales



Stratification increases (i.e. Pr increases)

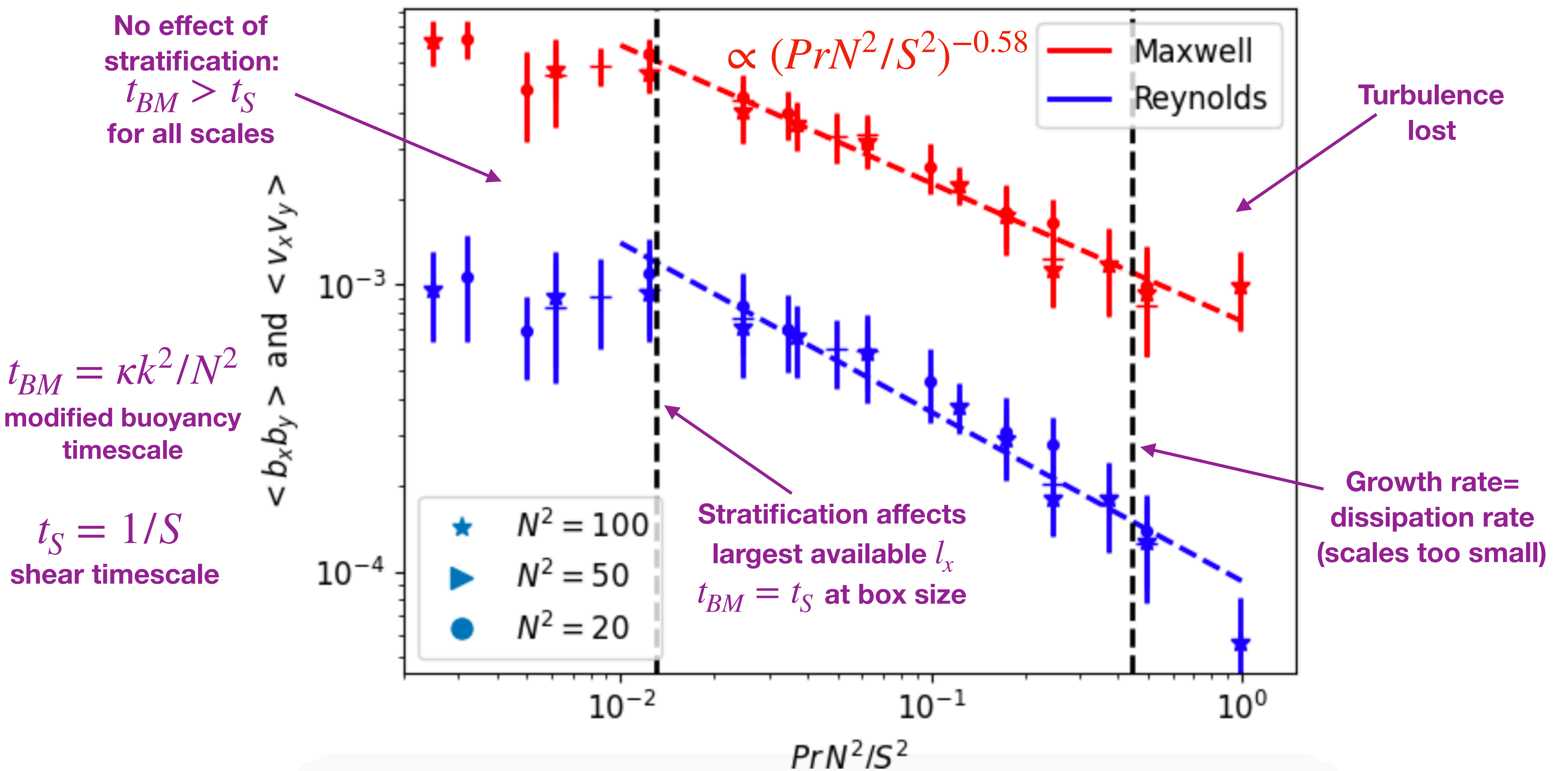
- ✓ Amplitude of radial velocity strongly reduced
- ✓ Small scales, especially in the z direction to optimise the growth rate despite stratification:
  $t_{eddies} = t_{shear} \implies l_z / v^{rms} = 1/S \implies l_z = v^{rms}/S$
- ✓ Longitudinal magnetic field only marginally affected

Growth rate from linear analysis of stratified MRI with $(k_z, k_x) >> k_y$

$$\sigma = \frac{S}{2} \frac{1}{\frac{N^2}{\kappa k_z^2} \frac{1}{4\Omega} + 1}$$

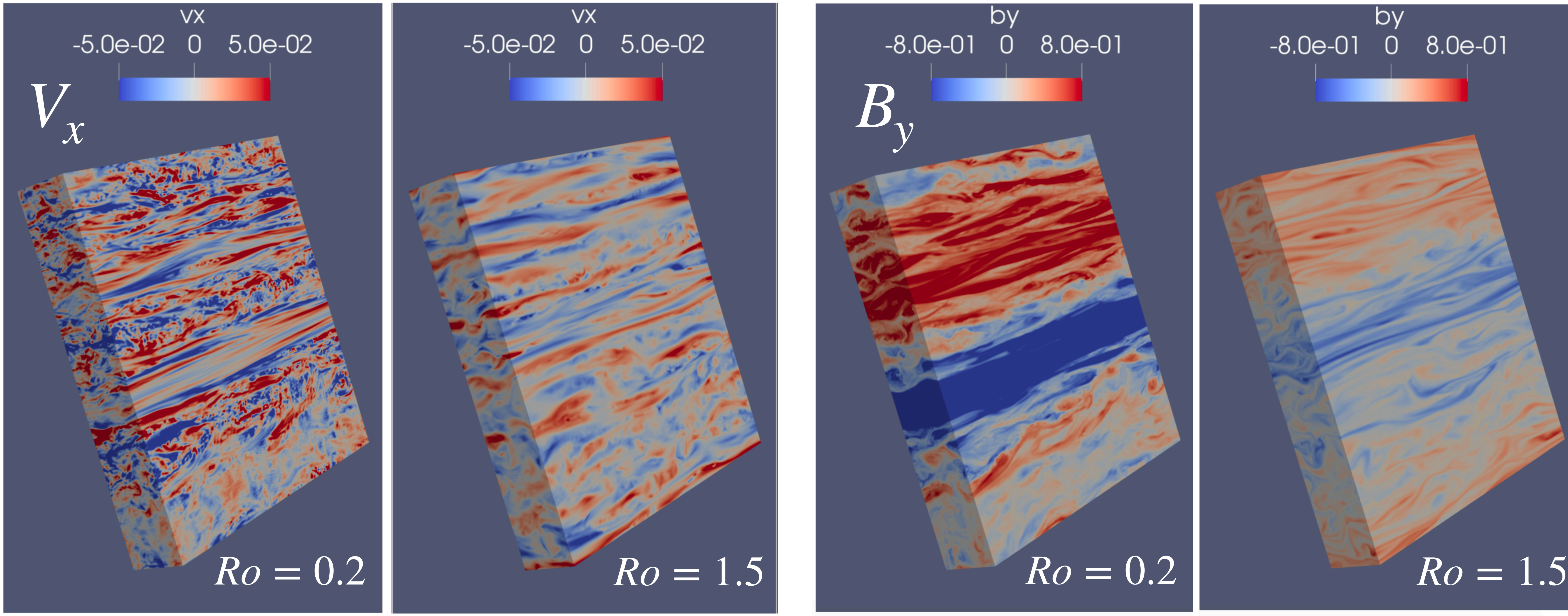
Impact of stratification: angular momentum transport

- ✓ Angular momentum conservation equation: $\frac{\partial R\rho v_\phi}{\partial t} + \rho \boldsymbol{u} \cdot \boldsymbol{\nabla}(\Omega R^2) + \boldsymbol{\nabla} \cdot \left[R\rho v_\phi \boldsymbol{u} - R\frac{B_\phi \boldsymbol{B}}{4\pi} \right] = 0$
- ✓ We are interested in the radial transport: $< v'_x v'_y >$ and $< b'_x b'_y >$
- ✓ Level of stratification controlled by N^2/κ (important role of thermal diffusivity)



Impact of rotation: amplitudes and lengthscales

Rotation measured by the Rossby number $Ro = S/\Omega$



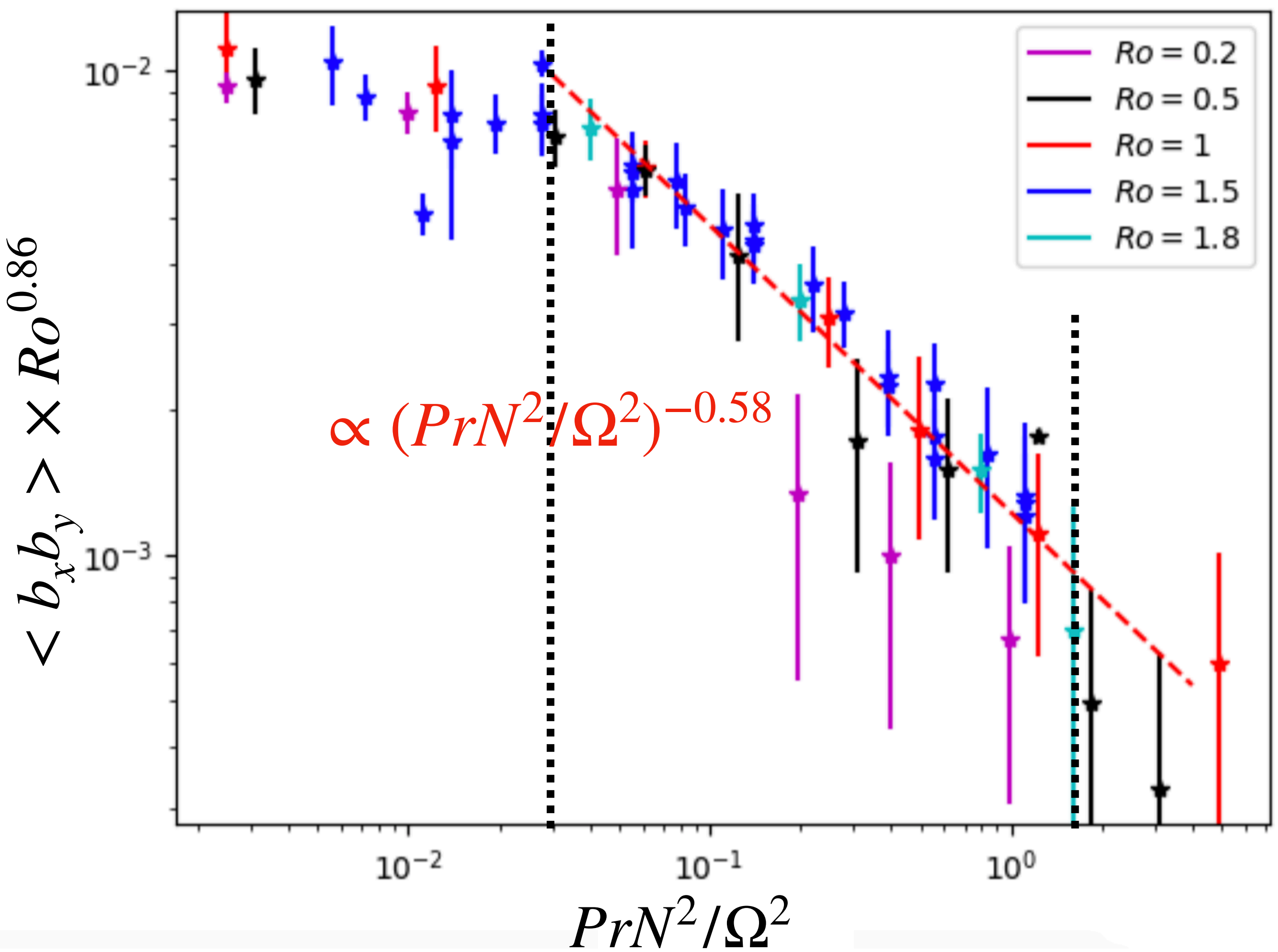
Linear analysis of stratified MRI

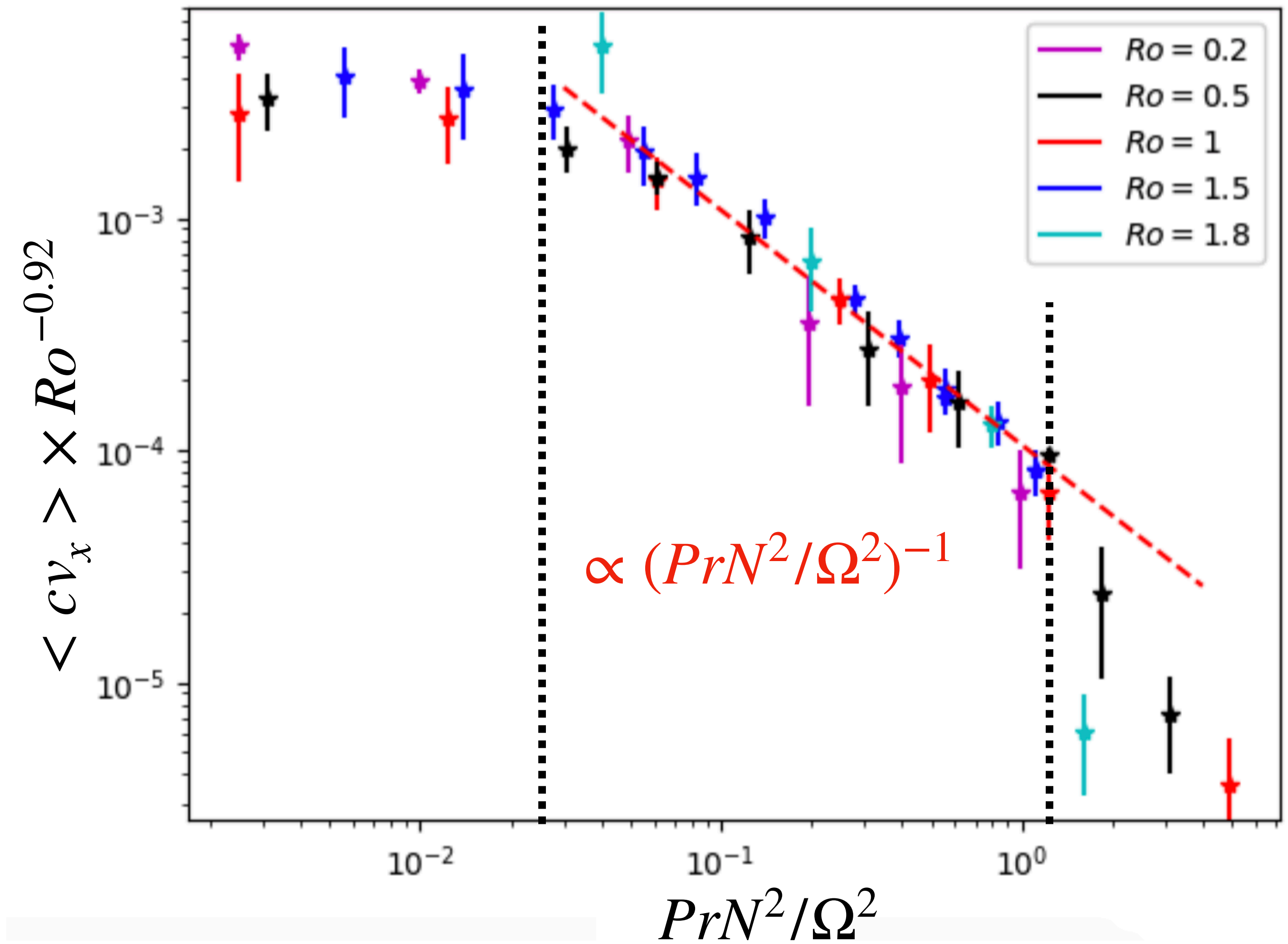
$$\sigma = \frac{S}{2}\frac{1}{\frac{N^2}{\kappa k_z^2}\frac{1}{4\Omega}+1} \quad \text{and} \quad \omega_A^2 = \Omega S \frac{1}{\frac{N^2}{\kappa S k_z^2}\frac{S}{4\Omega}+1}\left(1-\frac{S}{4\Omega}\frac{1}{\frac{N^2}{\kappa S k_z^2}\frac{S}{4\Omega}+1}\right)$$

- ✓ Decrease in velocity scale: turbulent eddies now constrained by rotation timescale $1/\Omega < 1/S$
- ✓ Increase in velocity amplitude: compatible with linear theory (larger growth rate for larger rotation rate => more energy injection)
- ✓ Increase in magnetic field amplitude: also compatible with linear theory (Alfvén frequency grows with rotation rate)

Combined effects of stratification and rotation on AM and chemical transport

✓ Combined effect of rotation and stratification on $< b_x b_y >$ and $< cv_x >$





✓ Transport less efficient on chemicals than AM

✓ Scaling of angular momentum different from chemical element transport for stratification: more severe impact on chemical elements

Summary/conclusions

✓ Transport by the magneto-rotational instability

- MRI dynamo cases found in stably stratified regimes
- No need for special initial field and no need for high shear
- Maxwell stress dominant: transport of AM different from chemical species
- 3D simu-based scaling with stratification and rotation available for MRI and ready to be reintroduced in 1D stellar evolution models

✓ We find scalings approximately such that:

- The turbulent transport coefficients are independent on viscosity in the relevant regime ($PrN^2/S^2 < 1,\ S/\Omega < 2,\ Rm > 10^5$)
- Turbulent mixing (on chemicals) independent on scale
- Maxwell stress seem to depend on a large-scale L: what is it in stars?

Could be related to latitudinal extension beyond which model of equatorial dynamics is no longer valid

$$\nu^t_{AM} = \frac{< b_x b_y >}{4\pi\rho S} \approx \alpha \frac{\Omega^2}{S^2} (\kappa Ri^{-1})^{0.5} (SL^2)^{0.5}$$

$$D_t = \frac{< c v_x >}{dC/dx} \approx \beta \frac{\Omega}{S} (\kappa Ri^{-1})$$

with $Ri = N^2/S^2$ and α and β of order 1

Jouve et al., A&A, under review