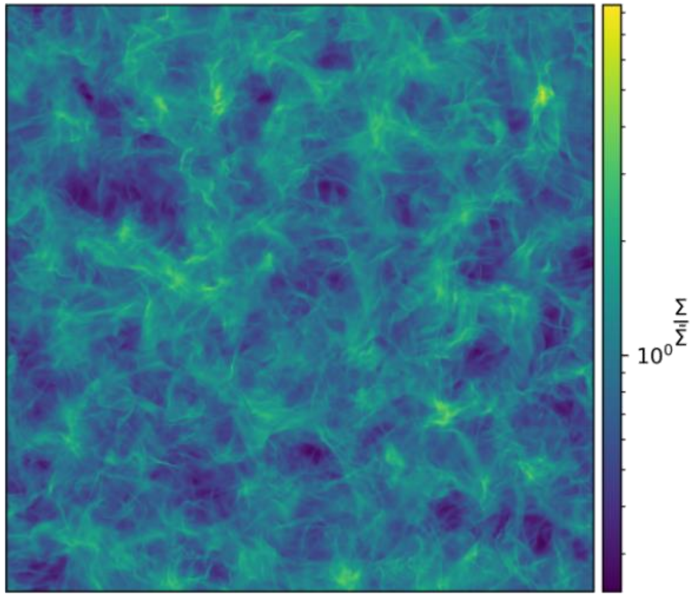
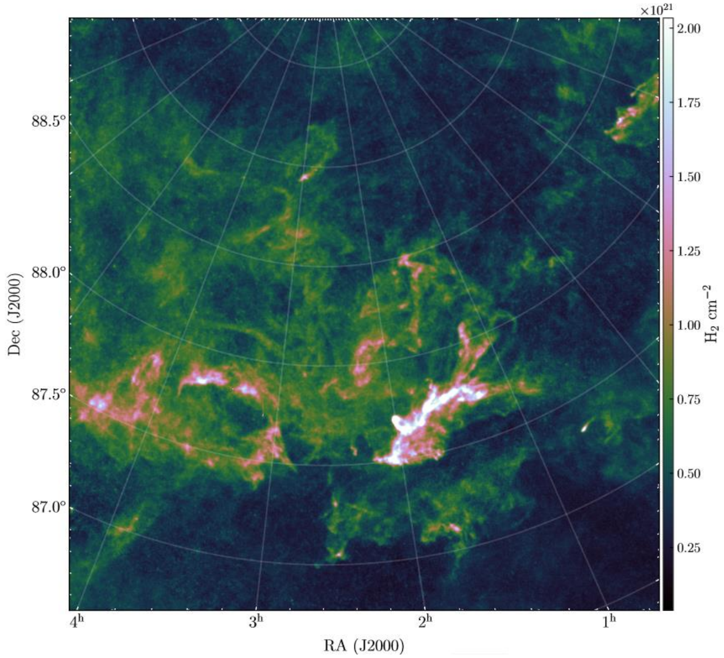
From a mass invariant in the compressible gravoturbulent ISM to the characteristic mass of the cores

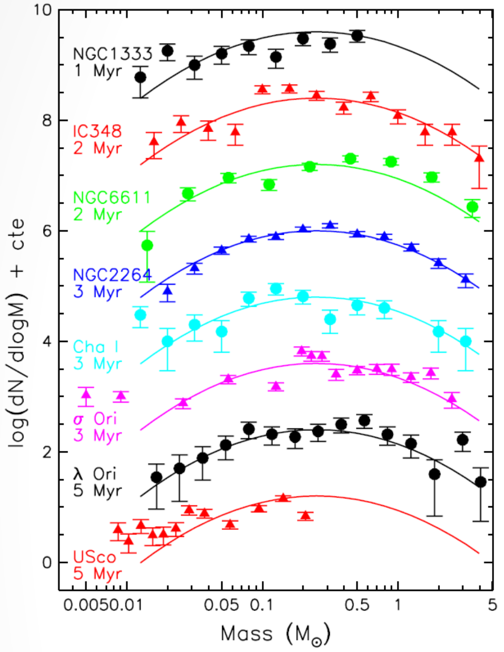




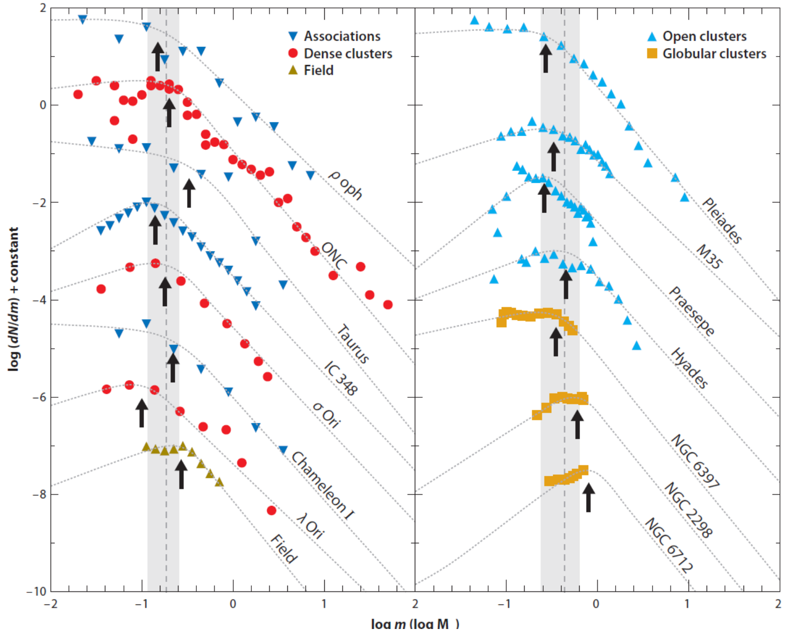
From Robitaille et al. 2019

Pierre DUMOND
CRAL – ENS Lyon
3rd year PhD
Supervisors: Jeremy Fensch and Gilles Chabrier

The observed characteristic mass of the cores



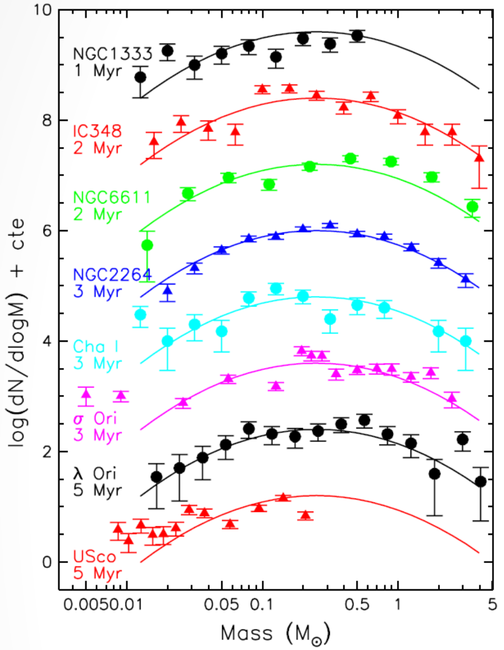
Offner+2014



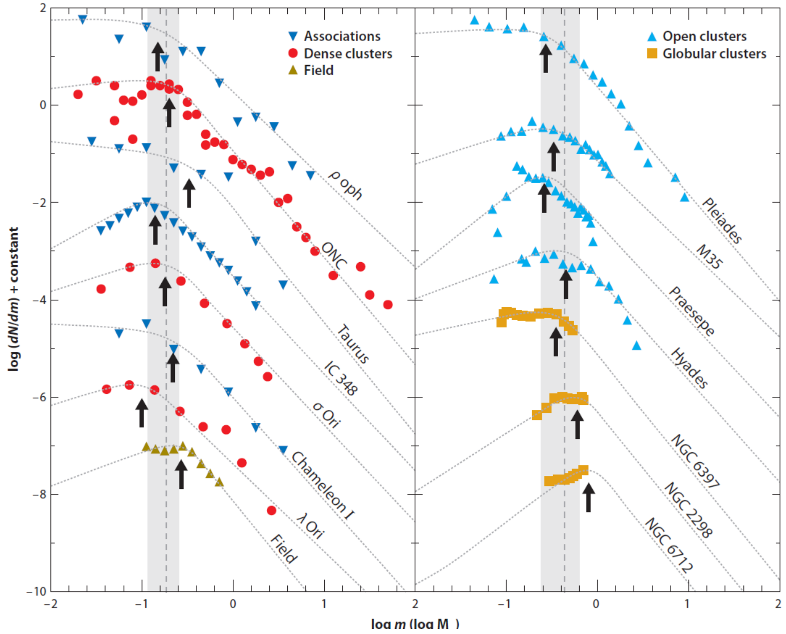
Bastian+2010

The IMF exhibits a characteristic mass close to $0.3M_\odot$
in Milky-Way like star forming regions

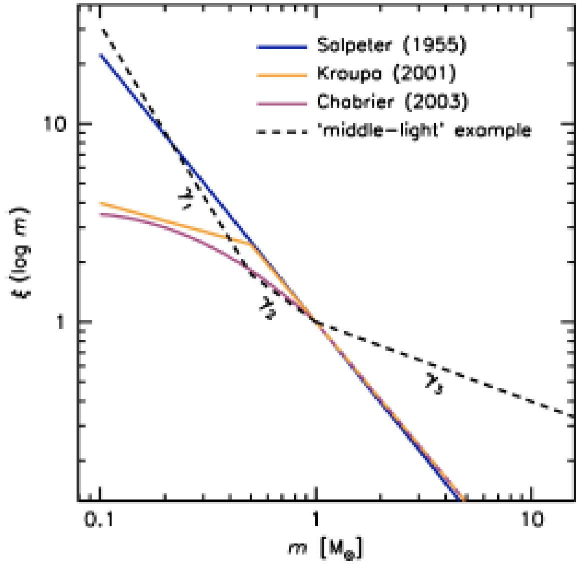
The observed characteristic mass of the cores



Offner+2014



Bastian+2010

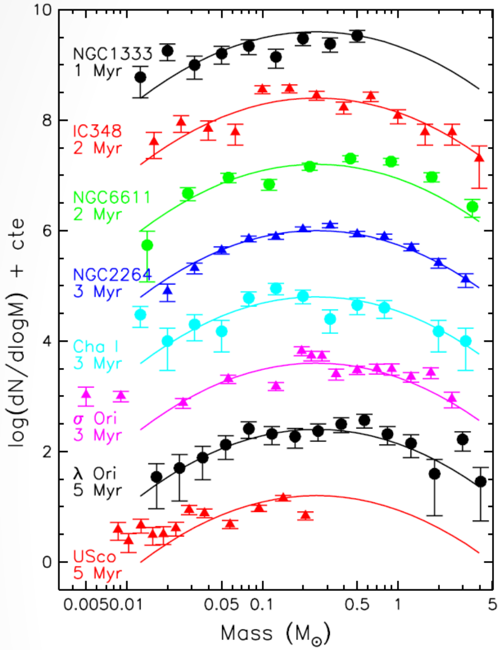


Early Type Galaxies ; Van Dokkum+2024

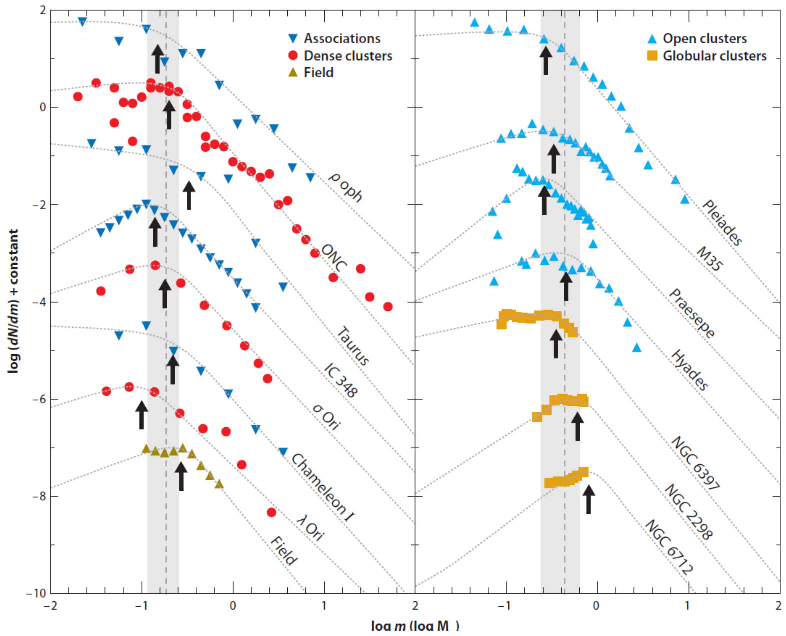
The IMF exhibits a characteristic mass close to $0.3M_\odot$ in Milky-Way like star forming regions

The characteristic mass may vary in extreme star forming regions

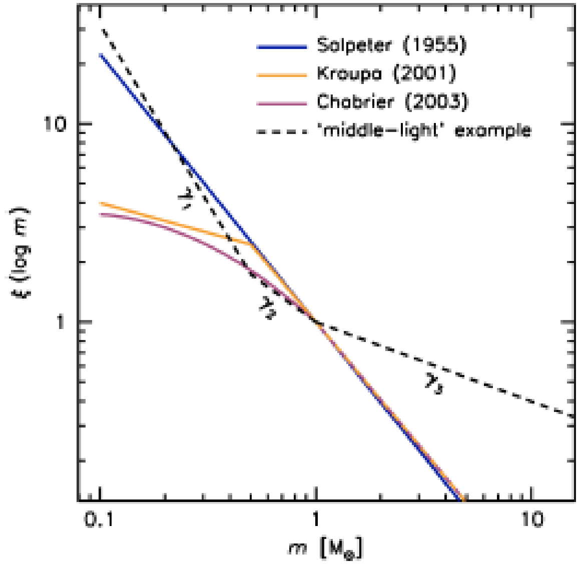
The observed characteristic mass of the cores



Offner+2014



Bastian+2010



Early Type Galaxies ; Van Dokkum+2024

The IMF exhibits a characteristic mass close to $0.3M_\odot$ in Milky-Way like star forming regions

The characteristic mass may vary in extreme star forming regions

From simulation of isolated cores, core-to-star formation efficiency $\in [0.3, 1]$ (Machida+2009, 2012) → Characteristic mass of the cores between 0,3 and $1M_\odot$ in MW

<u>Cores</u>: Mass reservoir for individual star or binaries (McKee & Tan, 2003)

How to explain the characteristic mass of the cores
both in MW-like and extreme star forming regions ?

How to explain the characteristic mass of the cores
both in MW-like and extreme star forming regions ?

Common interpretation of the characteristic mass

Jeans mass in molecular cloud (or Bonnor-Ebert mass)

$$M_\mathrm{J} = 2\ \mathrm{M}_\odot \left(\frac{c_s}{0{,}2\ \mathrm{km}\cdot\mathrm{s}^{-1}}\right)^3 \left(\frac{n}{10^3\ \mathrm{cm}^{-3}}\right)^{-1/2}$$

But:

- Not universal in MW (density can vary by orders of magnitude)

An invariant of compressible turbulence

- Turbulent invariant (Chandrasekhar 1951 & Jaupart & Chabrier 2021):
  - Assuming: statistical homogeneity and $\langle \rho(\boldsymbol{x})\, \rho(\boldsymbol{x}+\boldsymbol{q})\boldsymbol{v}(\boldsymbol{x}+\boldsymbol{q})\rangle_{\boldsymbol{x}}\ q^2 \xrightarrow[q\to\infty]{} 0$

$$\partial_t \rho + \vec{\nabla}(\rho\vec{v}) = 0 \implies \underbrace{M_{\text{inv}} = \bar{\rho}(t)\mathrm{Var}\left(\frac{\rho}{\bar{\rho}}\right) l_c^3 = C^{st}}_{\text{Temporal invariant}} \Leftrightarrow P_\rho(0) = C^{st}$$

Def: Correlation length: $l_c^\rho = \left(\frac{1}{8C_\rho(0)} \int_{\mathbb{R}^3} C_\rho(\boldsymbol{q})\mathrm{d}^3\boldsymbol{q}\right)^{1/3}$

Def: Autocovariance function: $C_\rho(\boldsymbol{q}) = \langle \rho(\boldsymbol{x})\, \rho(\boldsymbol{x}+\boldsymbol{q})\rangle_{\boldsymbol{x}} - \bar{\rho}^2$

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Def: Autocovariance function: $C_\rho(\boldsymbol{q}) = \langle \rho(\boldsymbol{x})\, \rho(\boldsymbol{x}+\boldsymbol{q}) \rangle_{\boldsymbol{x}} - \bar{\rho}^2$

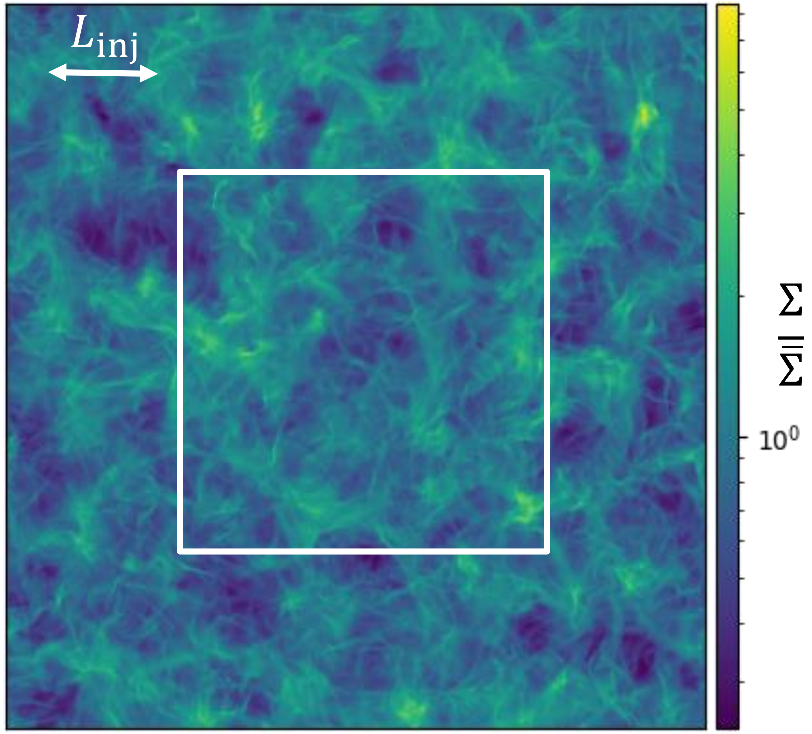
The conservation of M_{inv} involves only large scale processes:

Its time invariance will not be broken by any small scale processes

(local collapse, thermodynamics, magnetic field…)

Verification of the invariance of $M_{\rm inv}$

Projected density field



<u>Numerical setup</u>:

- RAMSES (fixed grid)
- Ornstein-Uhlenbeck turbulence forcing at $\frac{L_{\rm box}}{7}$
- Isothermal
- 2048^3

Often used to study the star formation process

Verification of the invariance of M_{inv}

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- Ornstein-Uhlenbeck turbulence forcing at $\frac{L_{\mathrm{box}}}{7}$
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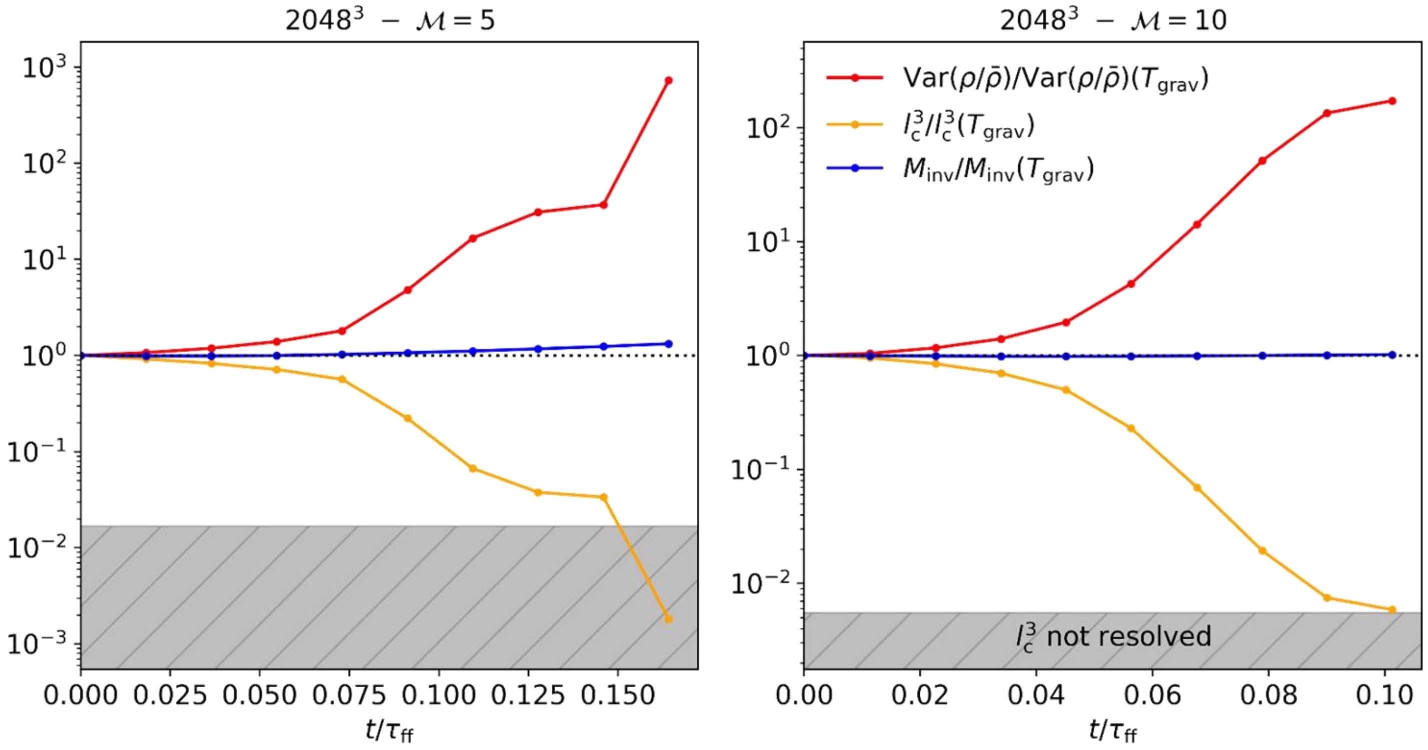
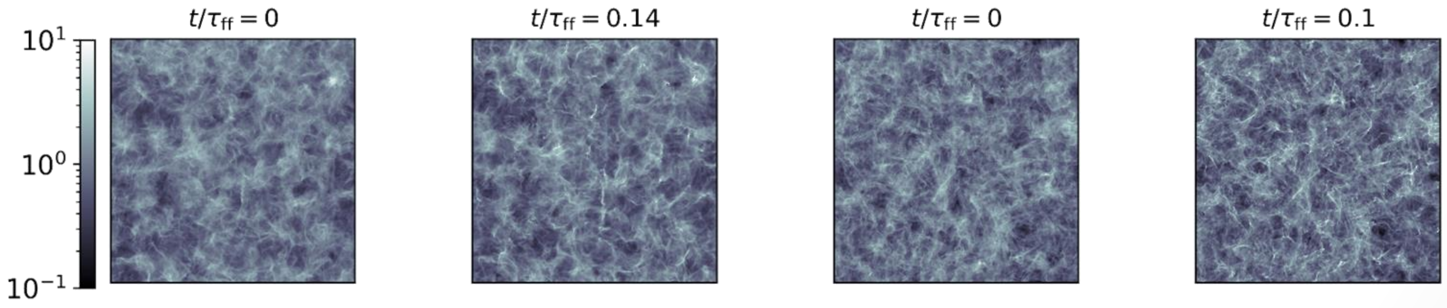
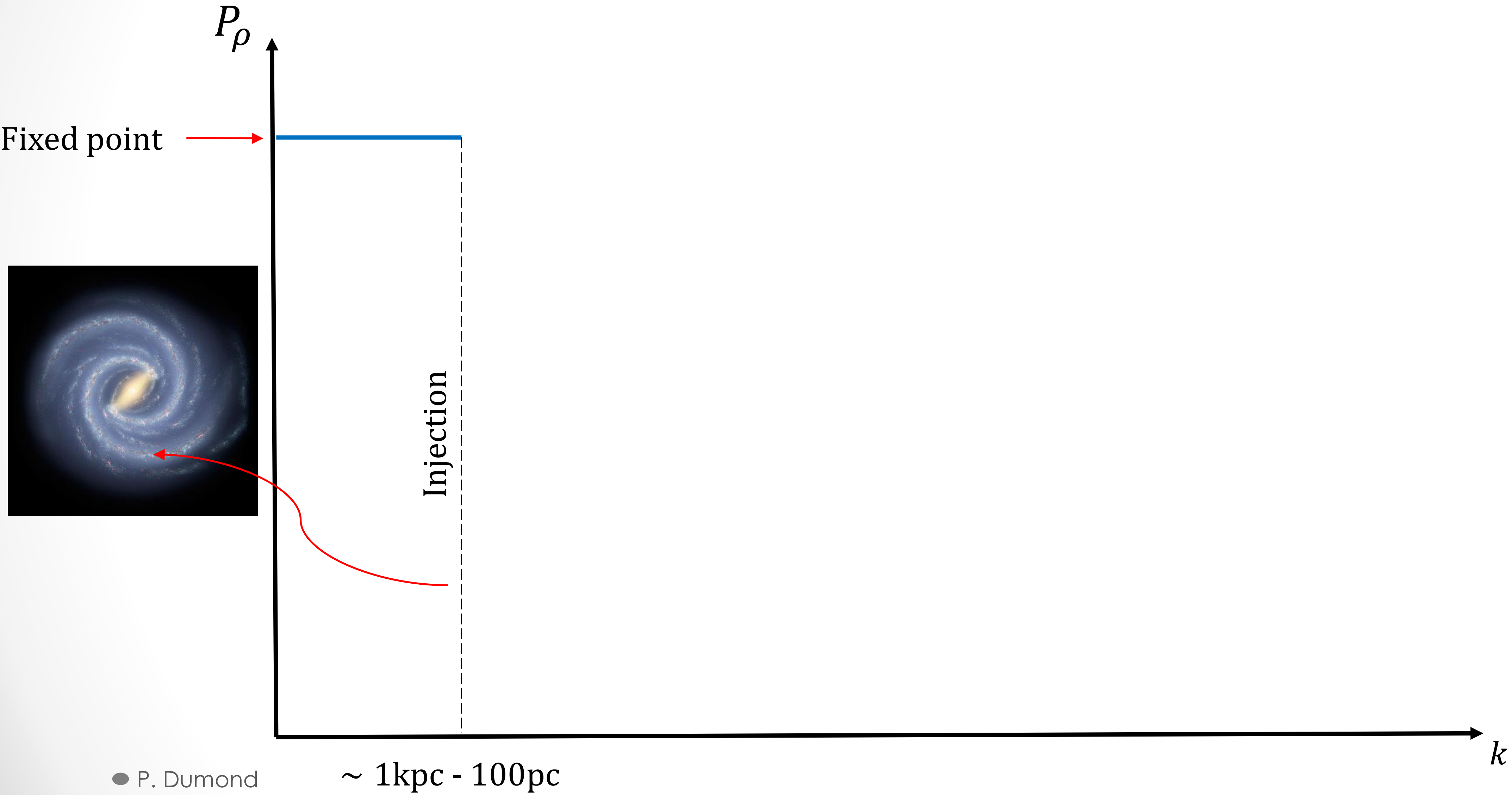




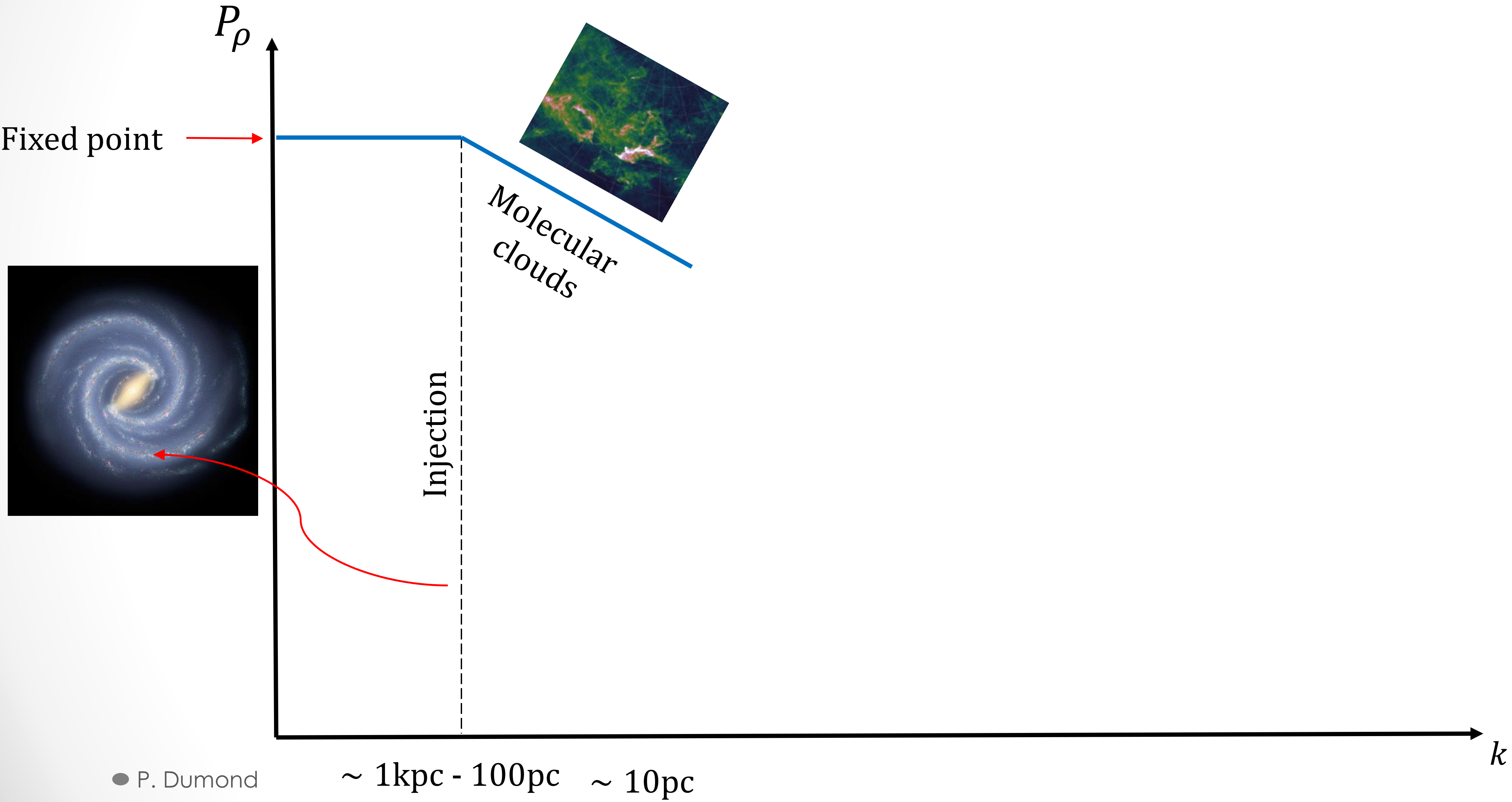
Dumond et al., subm.

Invariant with <u>self-gravity</u> verified ✓

Initial stages of star formation



Initial stages of star formation



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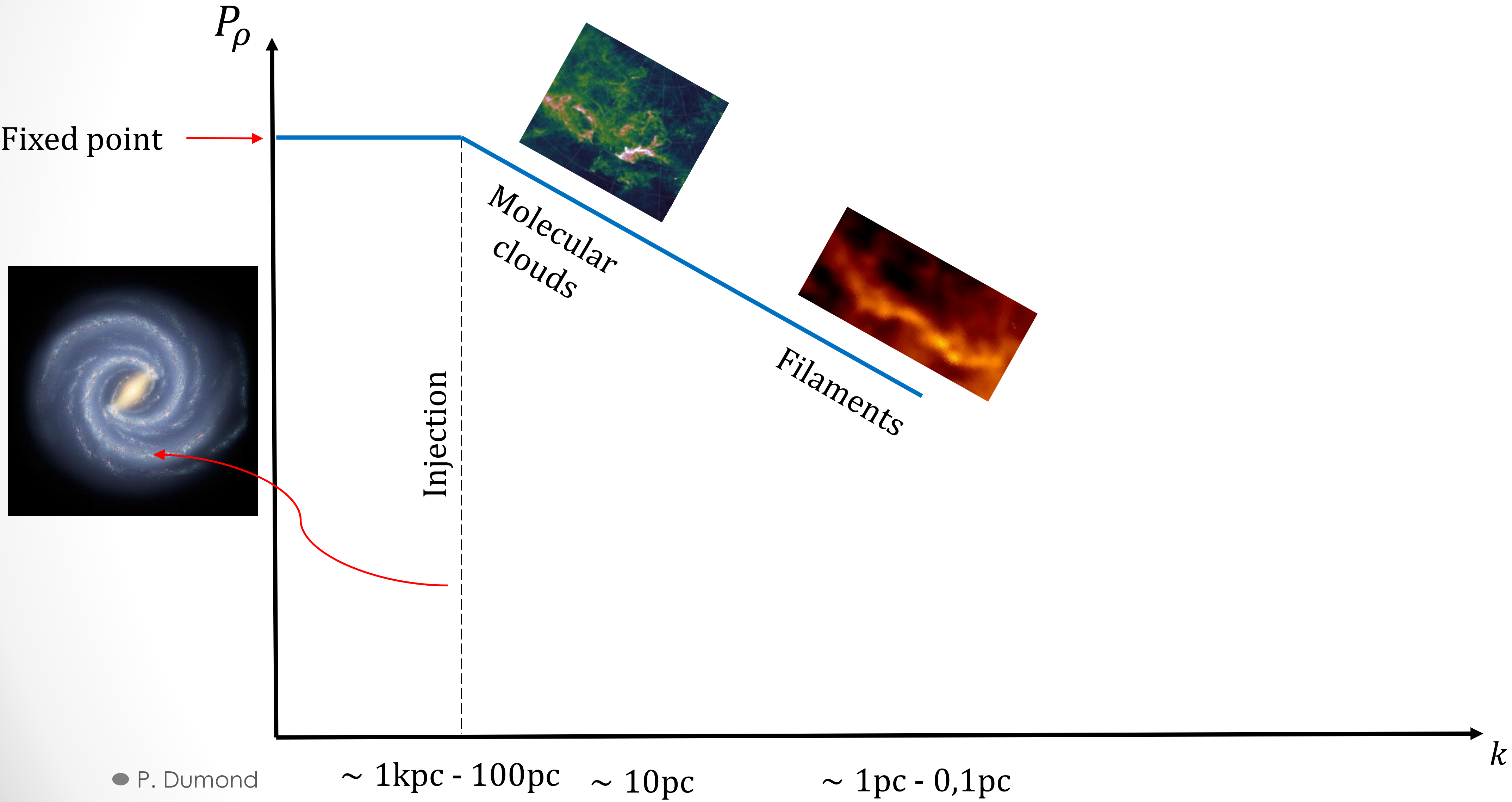

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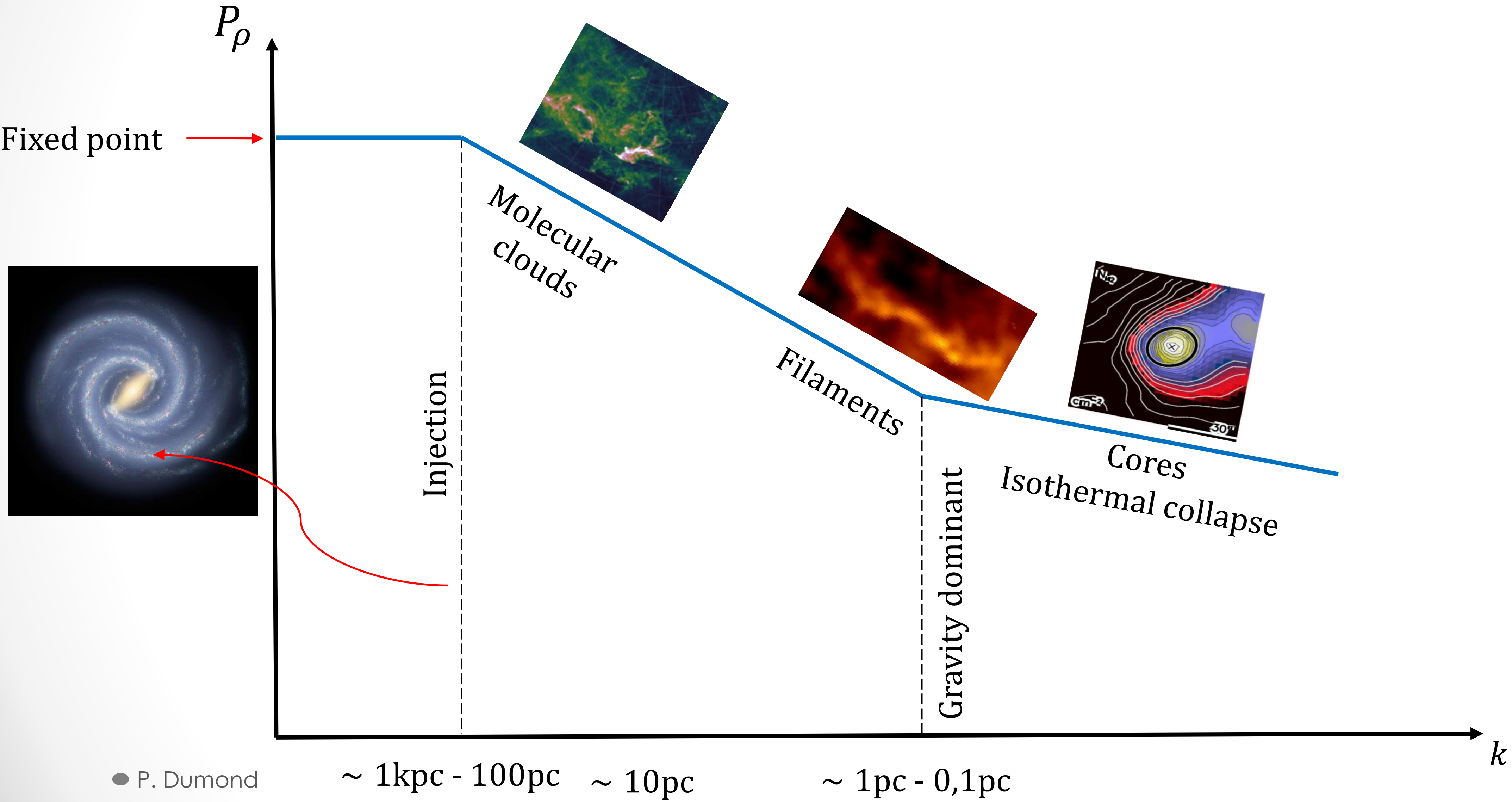

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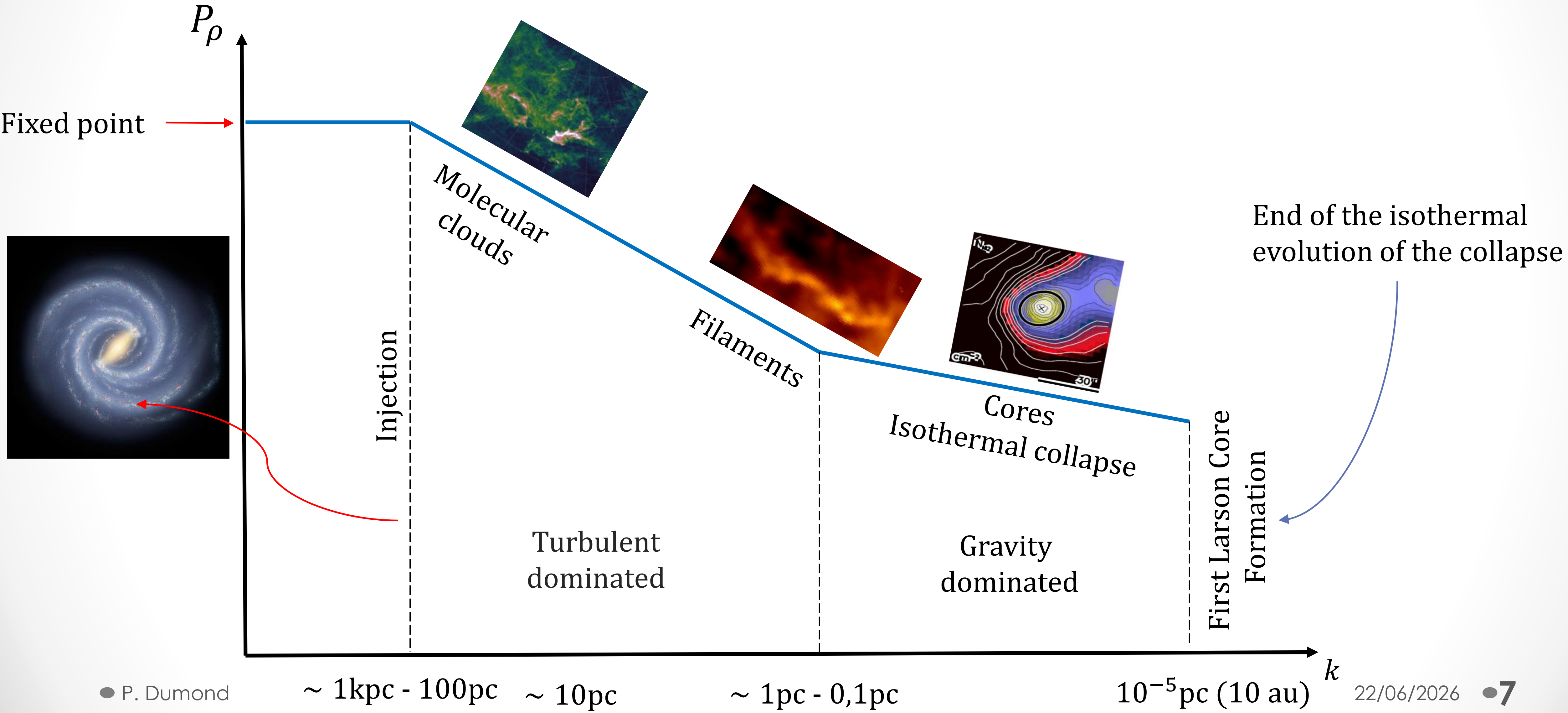


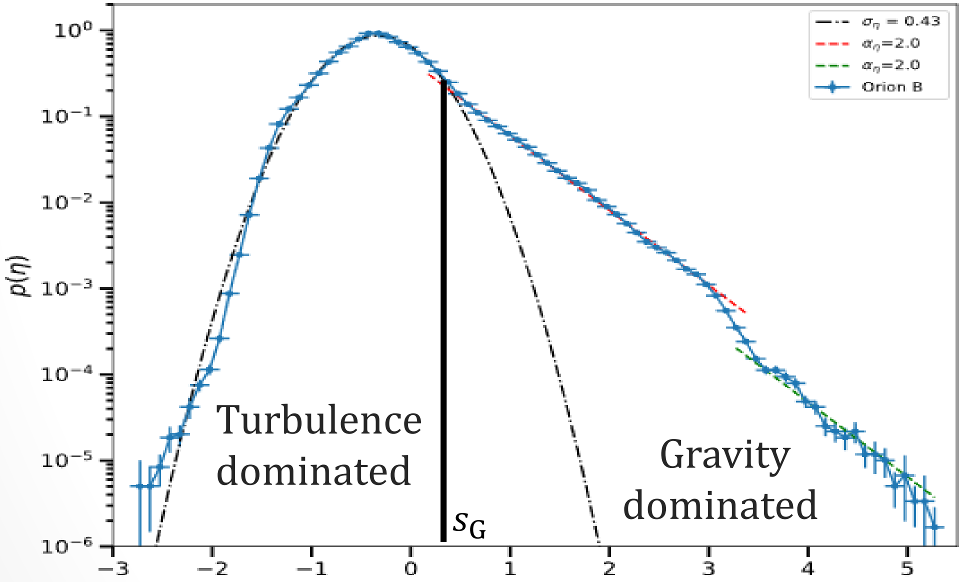
PDF model and transition scale

The density threshold

From Jaupart & Chabrier 2020:

$$|e^{s_{\mathrm{G}}} - 1| = (b\mathcal{M})^2 \frac{5\sigma_{\mathrm{v}}^2}{\pi G L_{\mathrm{c}}^2 \bar{\rho}} \left| \frac{s_{\mathrm{G}} + \frac{1}{2}\sigma_s^2}{\sigma_s^2} \right|$$

$$\text{with } s_{\mathrm{G}} = \ln\left(\frac{\rho_{\mathrm{G}}}{\bar{\rho}}\right)$$



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PDF model and transition scale

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$$|e^{s_G} - 1| = (b\mathcal{M})^2 \frac{5\sigma_v^2}{\pi G L_c^2 \bar{\rho}} \left| \frac{s_G + \frac{1}{2}\sigma_s^2}{\sigma_s^2} \right|$$

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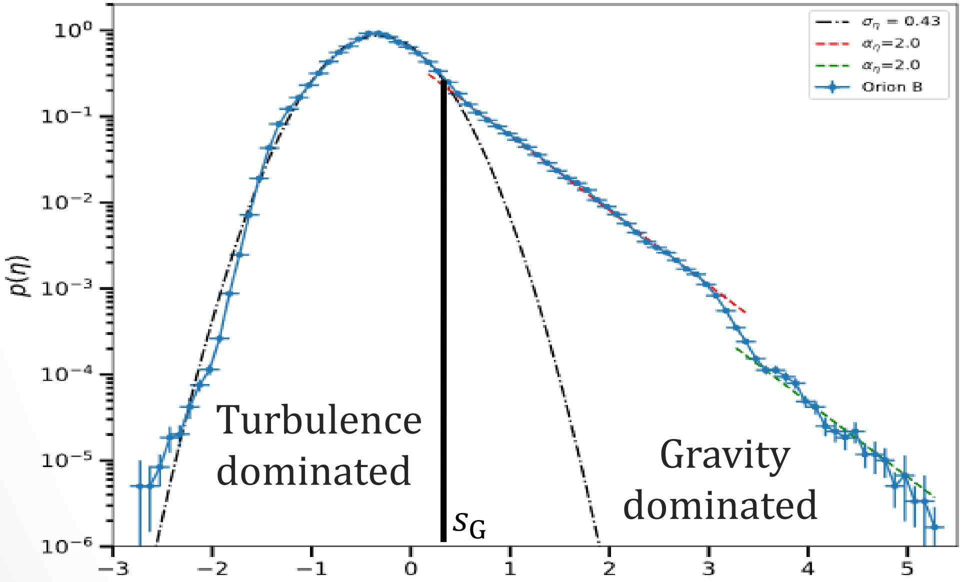

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The transition scale:

Jeans scale instability at the density ρ_G:

$$L_G = \frac{\sqrt{\pi} c_s}{\sqrt{\rho_G G}}$$

PDF model and transition scale

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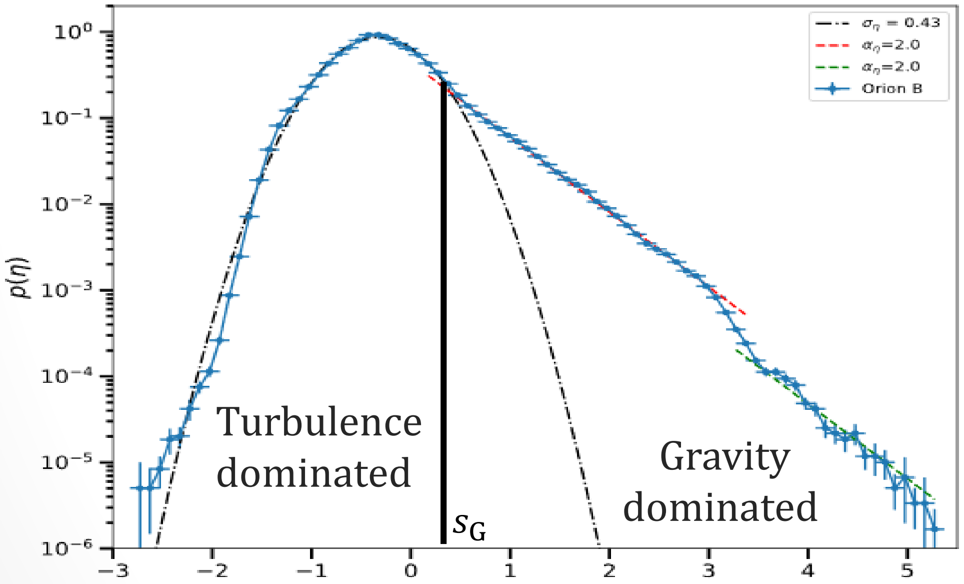


From Jaupart & Chabrier 2020

The transition scale:

Jeans scale instability at the density ρ_G:

$$L_\mathrm{G} = \frac{\sqrt{\pi} c_\mathrm{s}}{\sqrt{\rho_\mathrm{G} G}}$$

Approximate density threshold:

$$\rho_\mathrm{G} \simeq \rho_\mathrm{cloud}(b\mathcal{M})^2$$

Physically, density in the post-shock region.
Independent of the size of the cloud if it follows the Larson relations

$$\mathcal{M} \simeq \mathcal{M}_0 \left(\frac{L}{1\ \mathrm{pc}}\right)^{0,5}$$

$$\rho_\mathrm{cloud} \simeq d_0 \left(\frac{L}{1\ \mathrm{pc}}\right)^{-1}$$

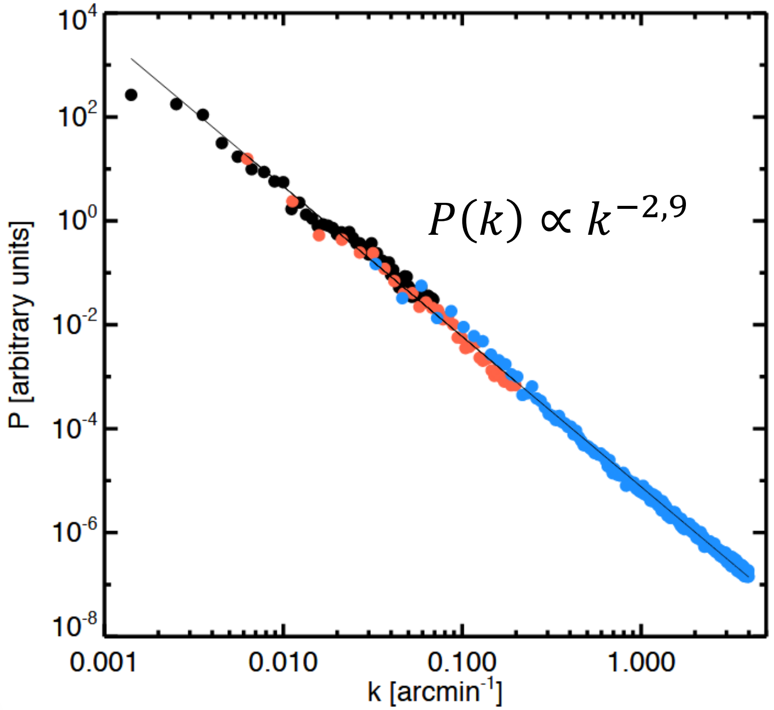
For typical ISM molecular clouds:

- $\rho_\mathrm{G} \sim 10^4 \mathrm{cm}^{-3}$
- $L_\mathrm{G} \sim 0{,}1\mathrm{pc}$

The slopes of the power spectrum

Turbulent dominated scales
$P(k) \propto k^{-3}$

Observed, see also (Dumond+2026)



Miville-Deschêne+2016

The slopes of the power spectrum

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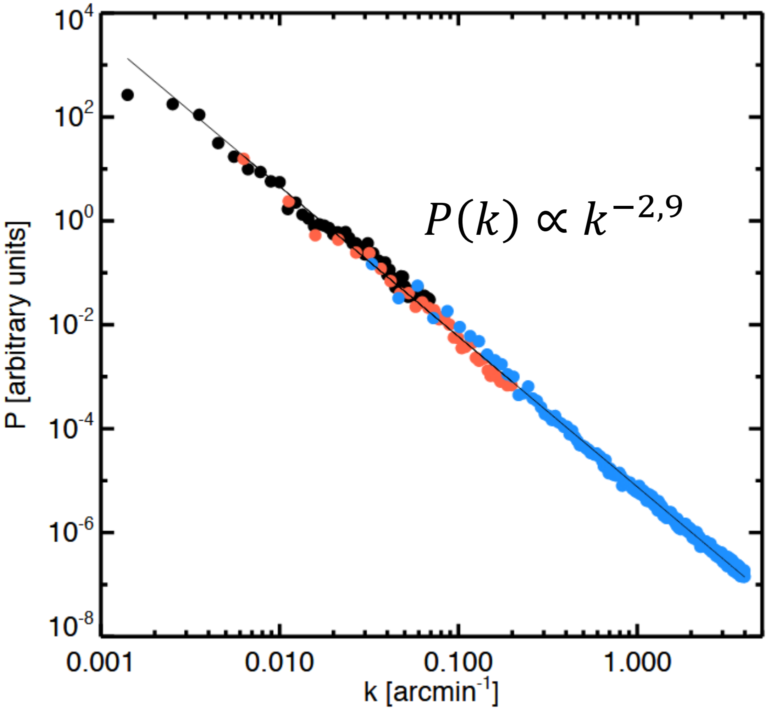


Miville-Deschêne+2016

Gravity dominated scales

Collapse of isothermal sphere: $\rho(r) \propto r^{-2}$
(Larson, Pentson, Shu)

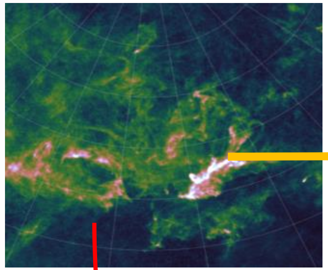
$\Rightarrow$

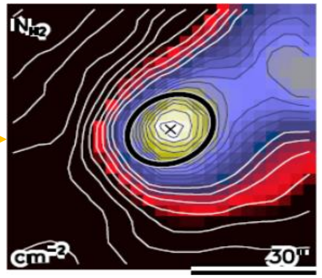
$P(k) \propto k^{-2}$

$P(k) \propto k^{-2}$ observed in actively star forming clouds: Taurus, Chamaeleon, W3
(see Federrath & Klessen 2013, Schneider+2022)

The characteristic mass of the cores in MW…

$$M_{\mathrm{cores}} = 1\mathrm{M}_\odot \left(\frac{\epsilon_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{0{,}07}\right)^{-3} \left(\frac{b\mathcal{M}_0}{0{,}75 \times 3}\right)^{6} \left(\frac{\rho_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{2 \times 10^4 \mathrm{cm}^{-3}}\right)^{-2} \left(\frac{d_0}{700\mathrm{cm}^{-3}}\right)^{3} \left(\frac{L_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{0{,}2\ \mathrm{pc}}\right)^{3}$$





Follow the Larson relations

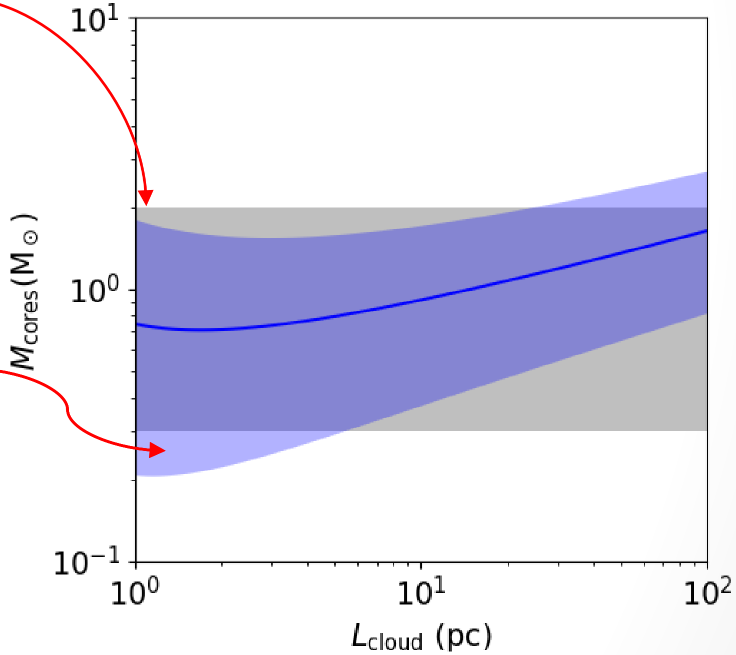
$$\mathcal{M} \simeq \mathcal{M}_0 \left(\frac{L}{1\ \mathrm{pc}}\right)^{0{,}5}$$

$$\rho_{\mathrm{cloud}} \simeq d_0 \left(\frac{L}{1\ \mathrm{pc}}\right)^{-1}$$

For MW-like star forming clouds: $\mathcal{M}_0 = 3$; $d_0 = 700\mathrm{cm}^{-3}$

Observed range of mean mass of the cores

Uncertainty of the model (b parameter)



Dumond et al., subm.

Independent on the injection mechanism, the galactic mean density and the size of star forming molecular cloud

...and in extreme star forming regions

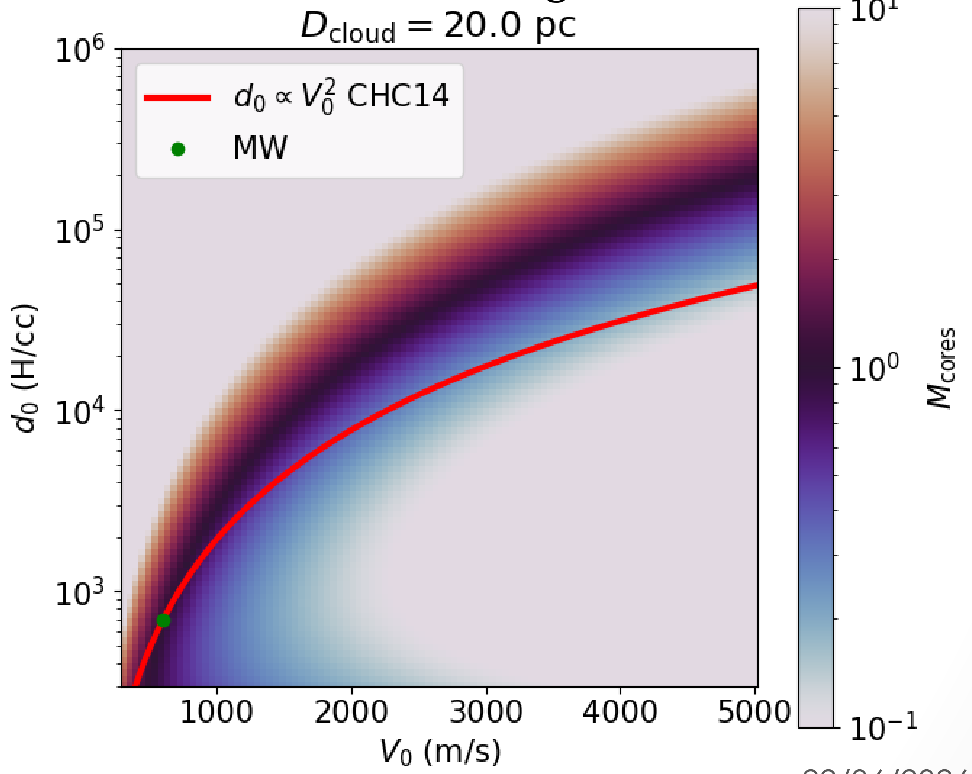
$$M_{\mathrm{cores}} = 1\mathrm{M}_\odot \left(\frac{\epsilon_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{0{,}07}\right)^{-3} \left(\frac{b\mathcal{M}_0}{0{,}75\times 3}\right)^{6} \left(\frac{\rho_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{2\times 10^4\mathrm{cm}^{-3}}\right)^{-2} \left(\frac{d_0}{700\mathrm{cm}^{-3}}\right)^{3} \left(\frac{L_{\mathrm{G}}(\mathcal{M}_0, d_0, L_{\mathrm{cloud}})}{0{,}2\ \mathrm{pc}}\right)^{3}$$

Chabrier et al. 2014: $\dfrac{d_0}{d_{\mathrm{MW}}} = \left(\dfrac{\mathcal{M}_0}{\mathcal{M}_{0_{\mathrm{MW}}}}\right)^2 = \left(\dfrac{P^{\mathrm{ext}}}{P^{\mathrm{ext}}_{\mathrm{MW}}}\right)^{1/2}$

Expected excess of low mass stars in extreme star forming regions (denser & more turbulent):

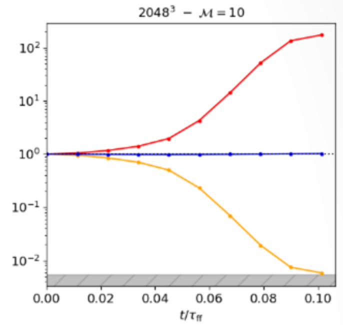
- Consistent with observations (Van Dokkum+, 2012, 2013, 2025) for Early type galaxies

Prediction of the evolution of the mass of the cores across star forming environments

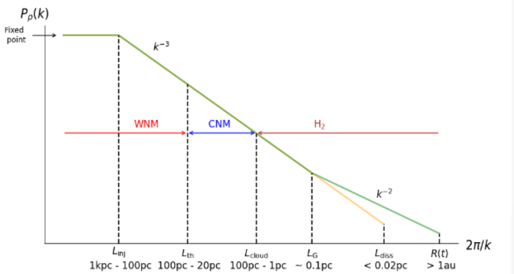


Dumond et al., subm.

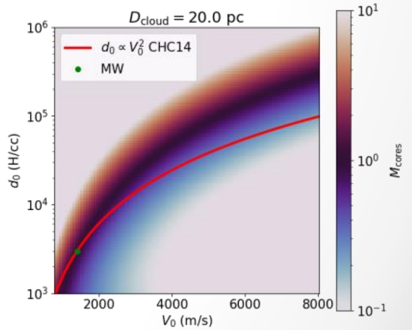
Conclusion



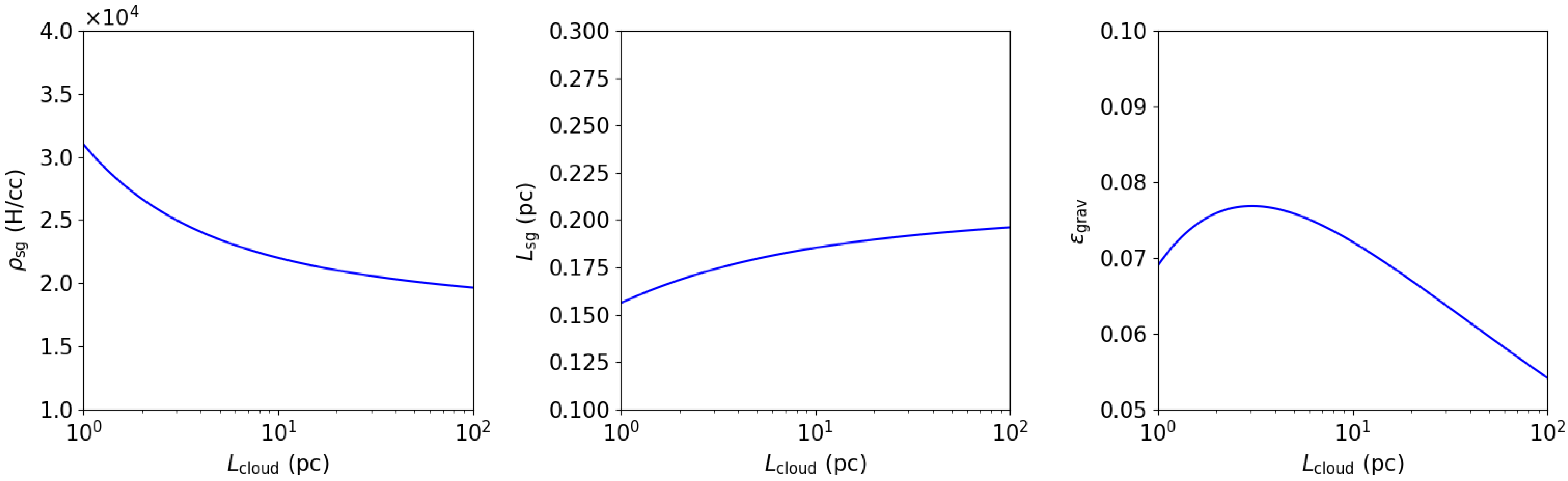
- Confirmation of invariance of M_{inv} in decaying and self-gravitating turbulence flows

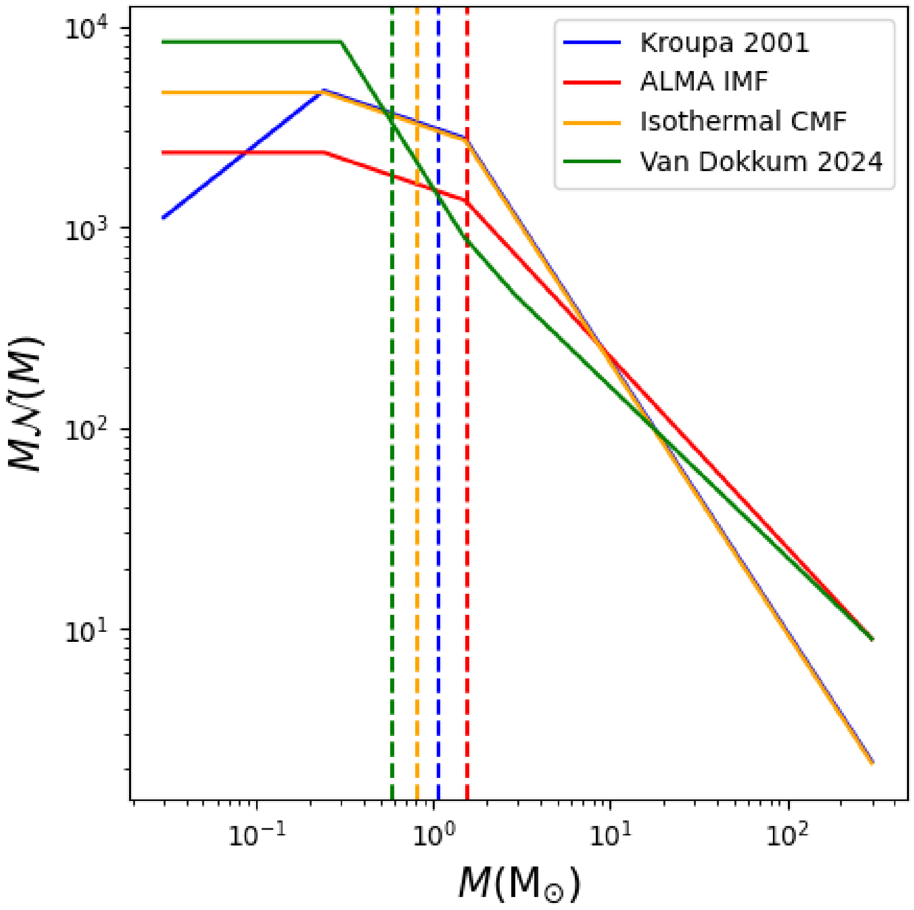


- Model of statistical properties of the gravoturbulent ISM from galactic scale to first Larson core scale:
  - Possible explanation of the mean mass of the prestellar cores both in MW like star forming regions and extreme environments
  - Both turbulent (at large scale) and gravity (at small scale) play a role in the characteristic mass and sufficient to explain it

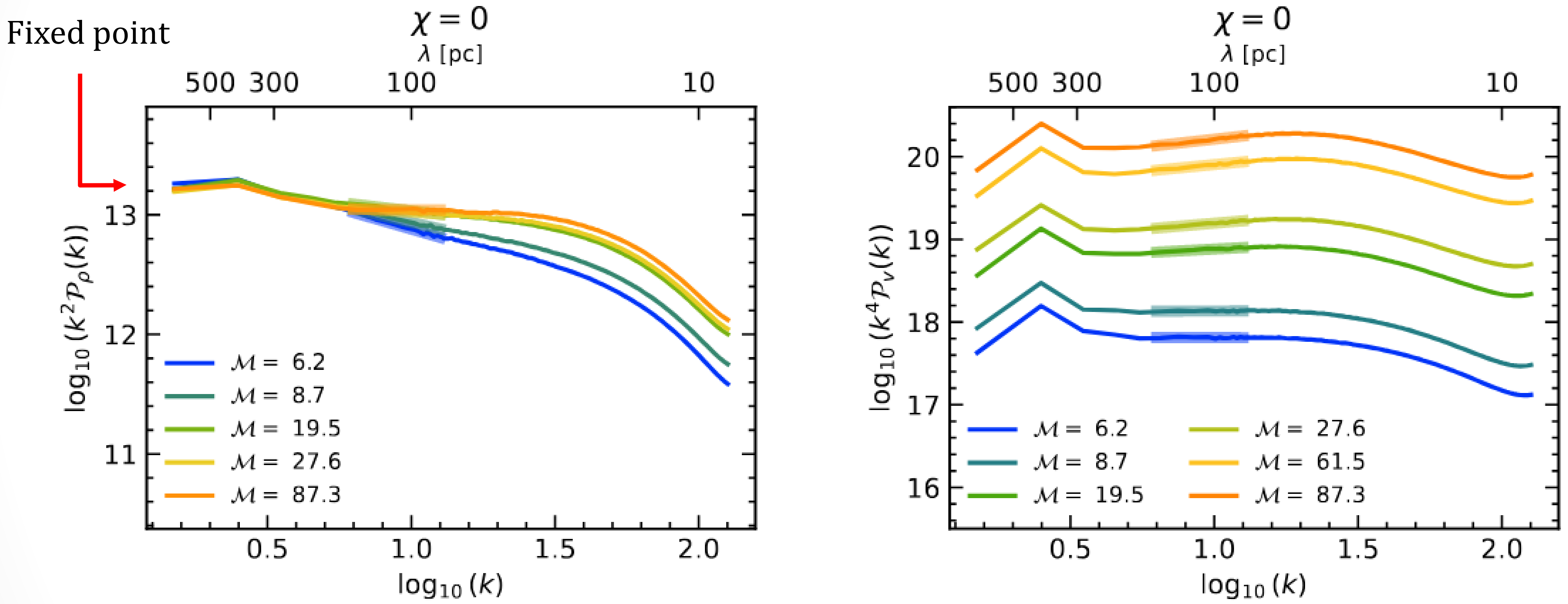


Thank you for your attention


×10^4
4.0
3.5
3.0
2.5
2.0
1.5
1.0
ρ_sg (H/cc)
10^0
10^1
10^2
L_cloud (pc)
0.300
0.275
0.250
0.225
0.200
0.175
0.150
0.125
0.100
L_sg (pc)
10^0
10^1
10^2
L_cloud (pc)
0.10
0.09
0.08
0.07
0.06
0.05
ε_grav
10^0
10^1
10^2
L_cloud (pc)


Kroupa 2001
ALMA IMF
Isothermal CMF
Van Dokkum 2024
10^4
10^3
10^2
10^1
$M\mathcal{N}(M)$
10^{-1}
10^0
10^1
10^2
$M(\mathrm{M}_\odot)$

Density vs. Velocity



Brucy+2024

Existence of the invariant only for the density field