Explaining the Schmidt-Kennicutt relation: a multi-scale analytical model

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What regulates star formation in galaxies?

-low (few percent) efficiency of star formation
-Schmidt-Kennicutt relation: $d\Sigma/dt \propto \Sigma^{1-1.4}$

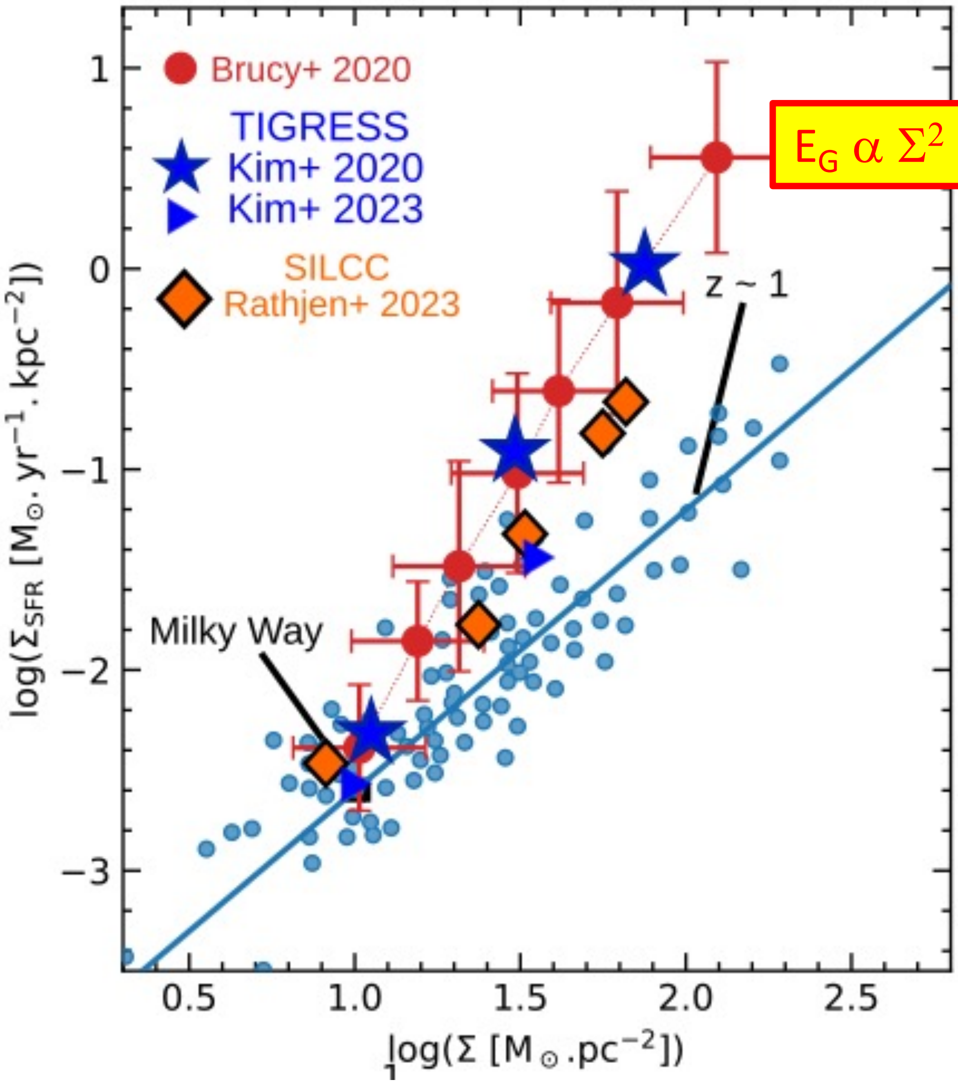
-Magnetic field
If strong enough, magnetic field counteract efficiently gravity, it then diffuses through ambipolar diffusion – Problem: field may not be strong enough
e.g. Mouschovias 1977, Shu+ 1987

-Turbulence
Dual role of turbulence which both compresses the gas through shocks and disperse the gas. Problem: turbulence decays in one freefall time
e.g. MacLow & Klessen 2004, H&Falgarone 2012

-Stellar Feedback
Likely important to inject turbulence in the ISM and to disrupt molecular clouds
Difficulty: A broad diversity of feedback and environments – hard to assess
e.g. Krumholz+2015, Girichidis+2020

Perhaps more fundamentally:
$E_G \propto \Sigma^2$ but $E_{feed} \propto \Sigma$ ($E_{feed} \propto \tau\_sfr * d\Sigma/dt = (\Sigma / d\Sigma/dt) * d\Sigma/dt$)

Trying to reproduce Schmidt-Kennicutt relation in kpc galactic boxes self-regulated ISM

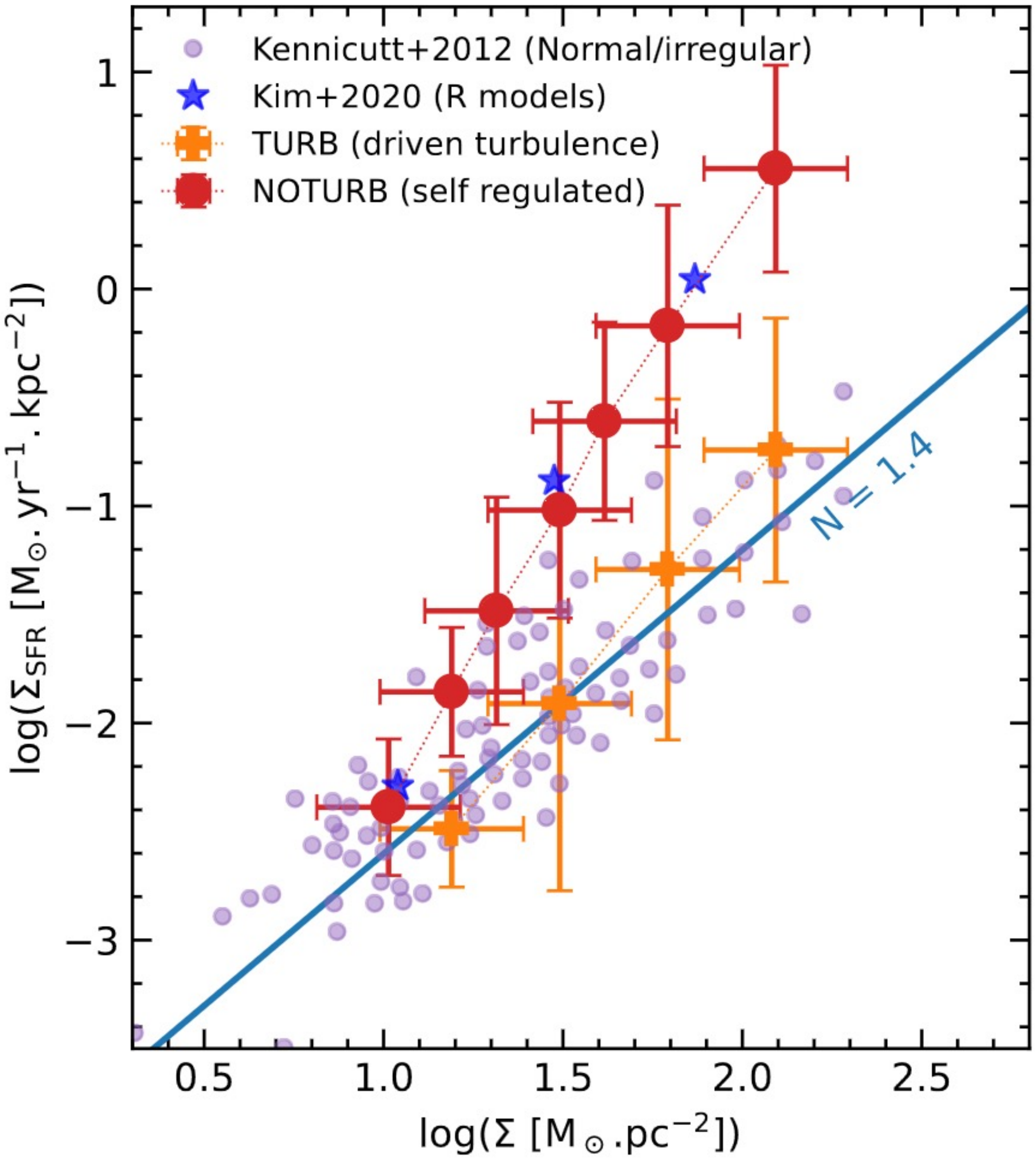


$E_G \alpha \Sigma^2$ but $E_{feed} \alpha \Sigma$

Brucy+2020,2023

Externally driven turbulence is able to explain Schmidt-Kennicutt (if sufficiently strong driving is applied…)

Brucy+2020, 2023



Kennicutt & Evans 2012

An analytical model to predict the star formation rate

(Press & Schecter 1974, H&Chabrier 2008)

Unstable mass at scale R from density PDF

$$M_{\rm tot}(R) = M_0 \int_{\delta_{\rm crit}(R)}^{\infty} e^{\delta} \mathcal{P}_R(\delta) {\rm d}\delta.$$

Unstable mass at scale R from cloud spectrum

$$M_{\rm tot}(R) = \int_0^{M_{\rm crit}(R)} M \mathcal{N}(M)\, {\rm d}M.$$

M_{crit}, ρ_{crit} from virial analysis:

$$M_{\rm crit}(R) = \frac{a_v}{G} R \left(3c_s^2 + \sigma_0 \left(\frac{R}{R_0}\right)^{2\eta_v} \right).$$

Taking the derivative with respect to R:

$$\mathcal{N}(M_{\rm crit}) = \mathcal{N}_1 + \mathcal{N}_2 = \tag{26}$$

$$\frac{\rho_0}{M_{\rm crit}} \frac{{\rm d}R}{{\rm d}M_{\rm crit}} \times \left(-\frac{{\rm d}\delta_{\rm crit}}{{\rm d}R} \exp(\delta_{\rm crit}) \mathcal{P}_R(\delta_{\rm crit}) + \int_{\delta_{\rm crit}}^{\infty} \exp(\delta) \frac{{\rm d}\mathcal{P}_R}{{\rm d}R} {\rm d}\delta \right),$$

Summing over the unstable cores divided by the replenishment time

$$\mathrm{SFR_{ff}} = \mathrm{SFR} \frac{\tau_{\rm ff,0}}{\rho_0 L_0^3} = \int_0^{\tilde{M}_{\rm sup}} \frac{\tilde{\mathcal{N}}(\tilde{M})\tilde{M}}{\tilde{\tau}_{\rm cont}(R)} {\rm d}\tilde{M},$$

We get the SFR as a function of Mach number, density PDF, density variance.

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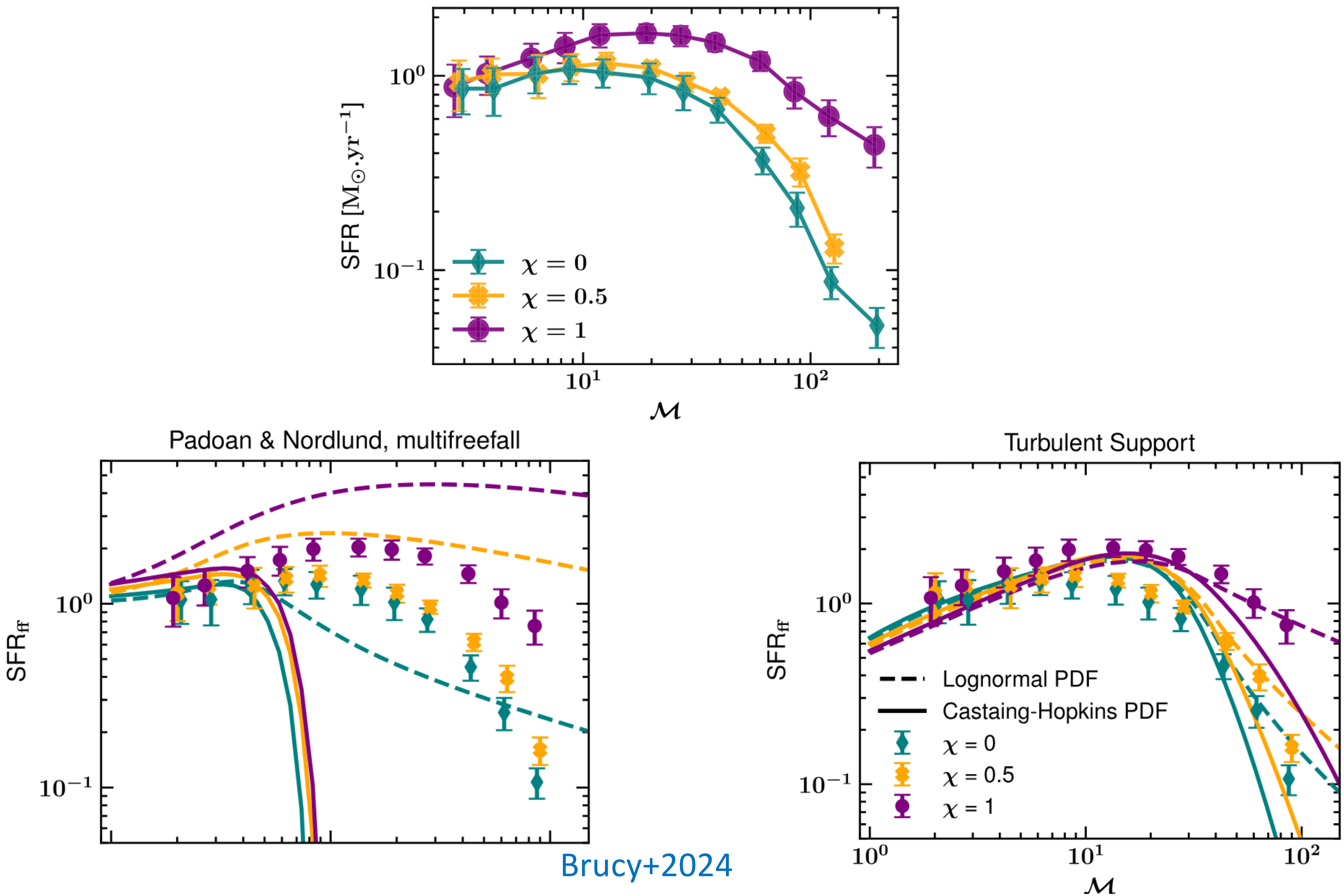
SFR from idealised (isothermal/no feedback) simulations
Comparison with analytical models



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Building a *complete* analytical model to get the Schmidt-Kennicutt relation

What do we need ?

Velocity dispersion => turbulent cascade – which sources?

Large scale gravitationnal instabilities and supernovae

A mean density => a column density and a scale height

Vertical equilibrium from gravity and turbulence

A magnetic intensity => assume dynamo and saturation proportional to the total kinetic energy

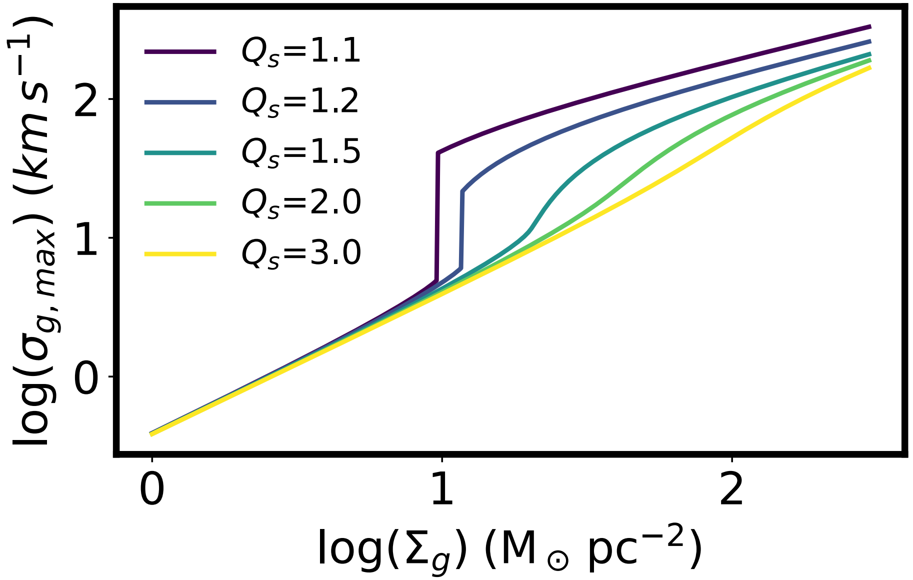
Assume a composite *critical* disk made of gas and stars

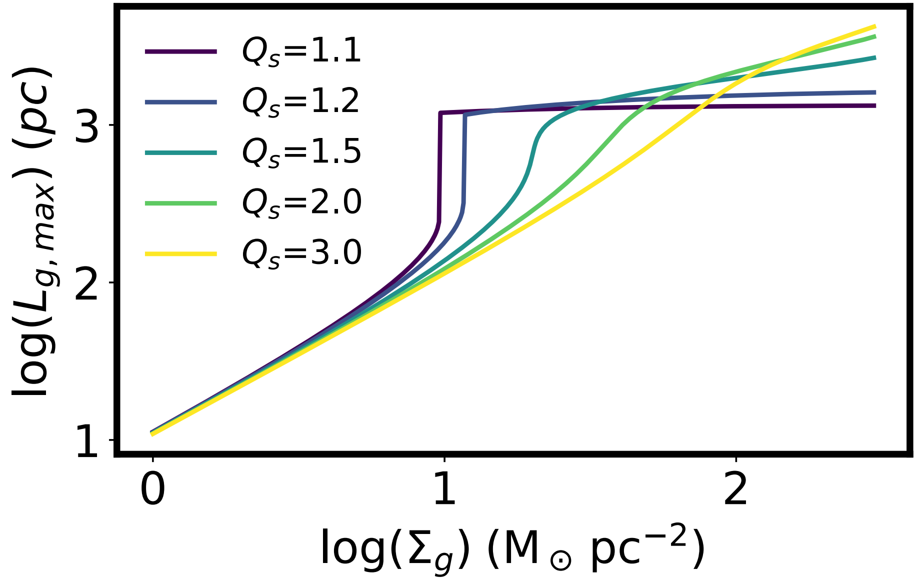
A Toomre criteria, Q*, is assumed
Gas + stars assumed to be critical

$$\omega^4 - \omega^2(\alpha_s + \alpha_g) + (\alpha_s\alpha_g - \beta_s\beta_g) = 0,$$

$$\begin{aligned}\kappa^2 + k^2C_s^2 - 2\pi Gk\Sigma_{s,0} &= \alpha_s,\\ \kappa^2 + k^2C_g^2 - 2\pi Gk\Sigma_{g,0} &= \alpha_g,\\ 2\pi Gk\Sigma_{s,0} &= \beta_s,\\ 2\pi Gk\Sigma_{g,0} &= \beta_g.\end{aligned}$$

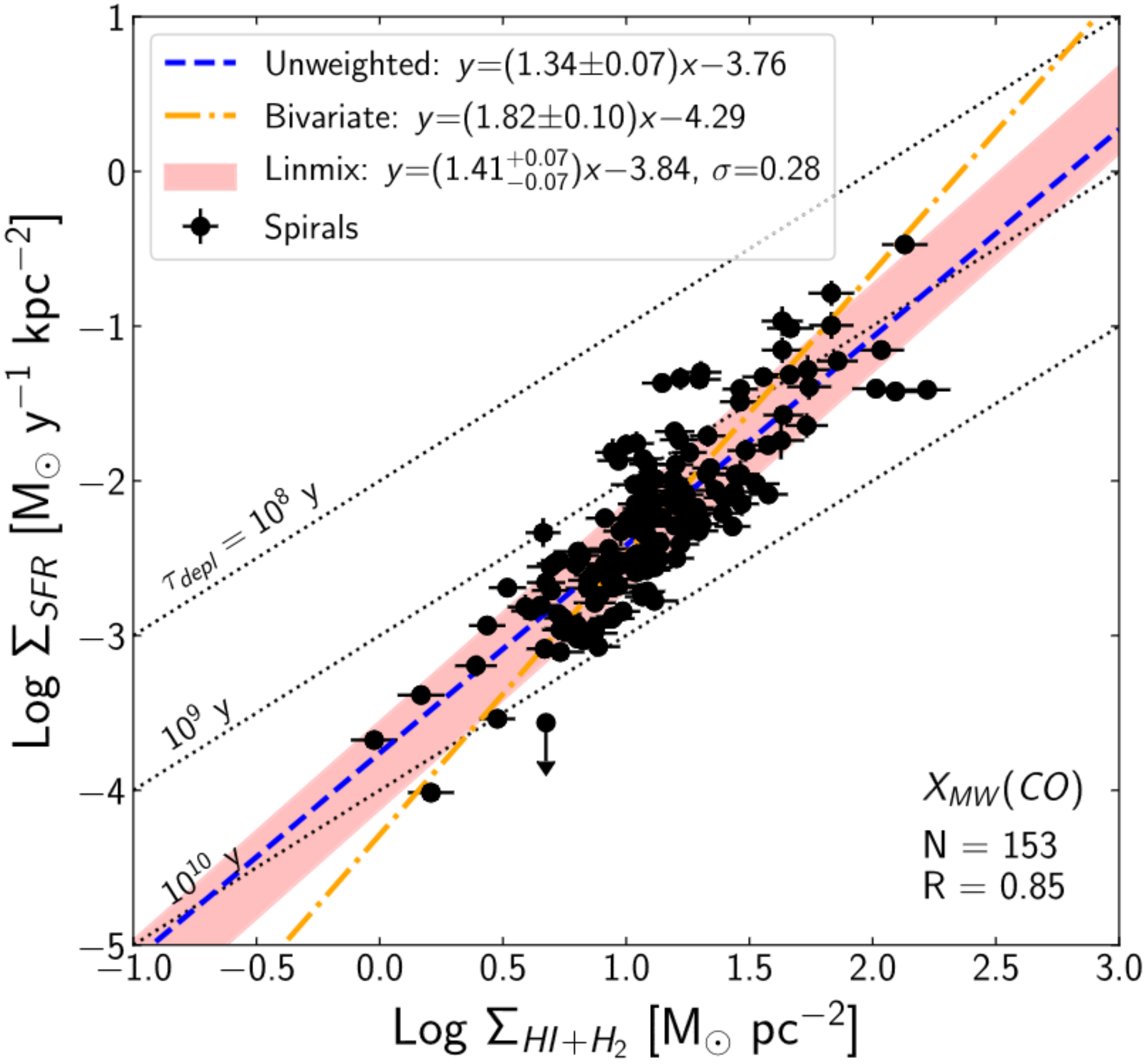
$$\sigma_g(k)^2 = 2\pi Gk^{-1}\Sigma_{g,0} - \kappa^2k^{-2} + \frac{4\pi^2G^2\Sigma_{s,0}\Sigma_{g,0}}{\kappa^2 + k^2\sigma_{s,0}^2 - 2\pi Gk\Sigma_{s,0}}.$$





Jog&Solomon 1984

A data sample from De Los Reyes & Kennicutt 2019

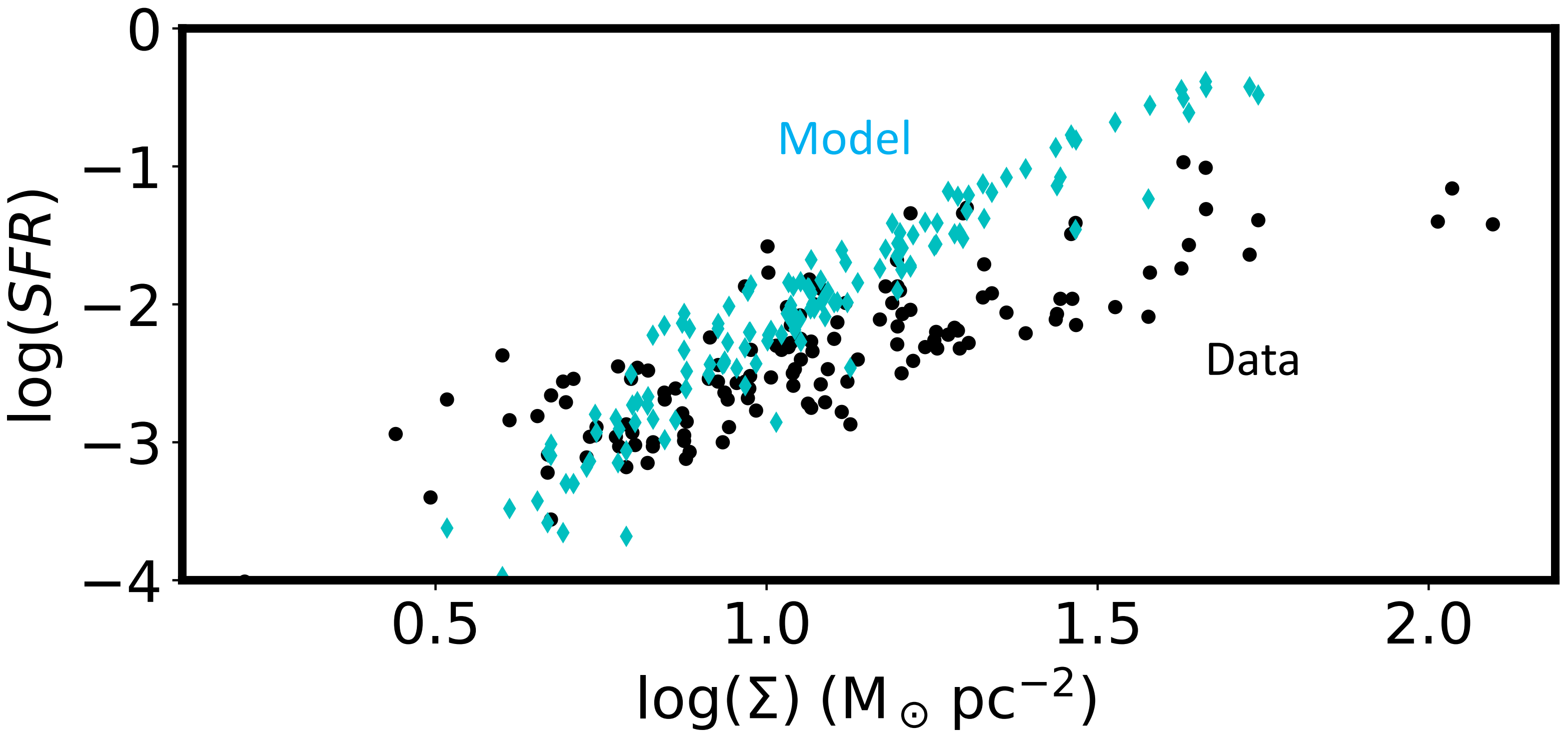


Comparison with a model that is too efficient

For each data point (De Los Reyes & Kennicutt 2019), we have:

- -gas column density
- -stellar column density
- -rotation

Hydrodynamical model, broad log-normal density PDF, $Q_*=1.5$

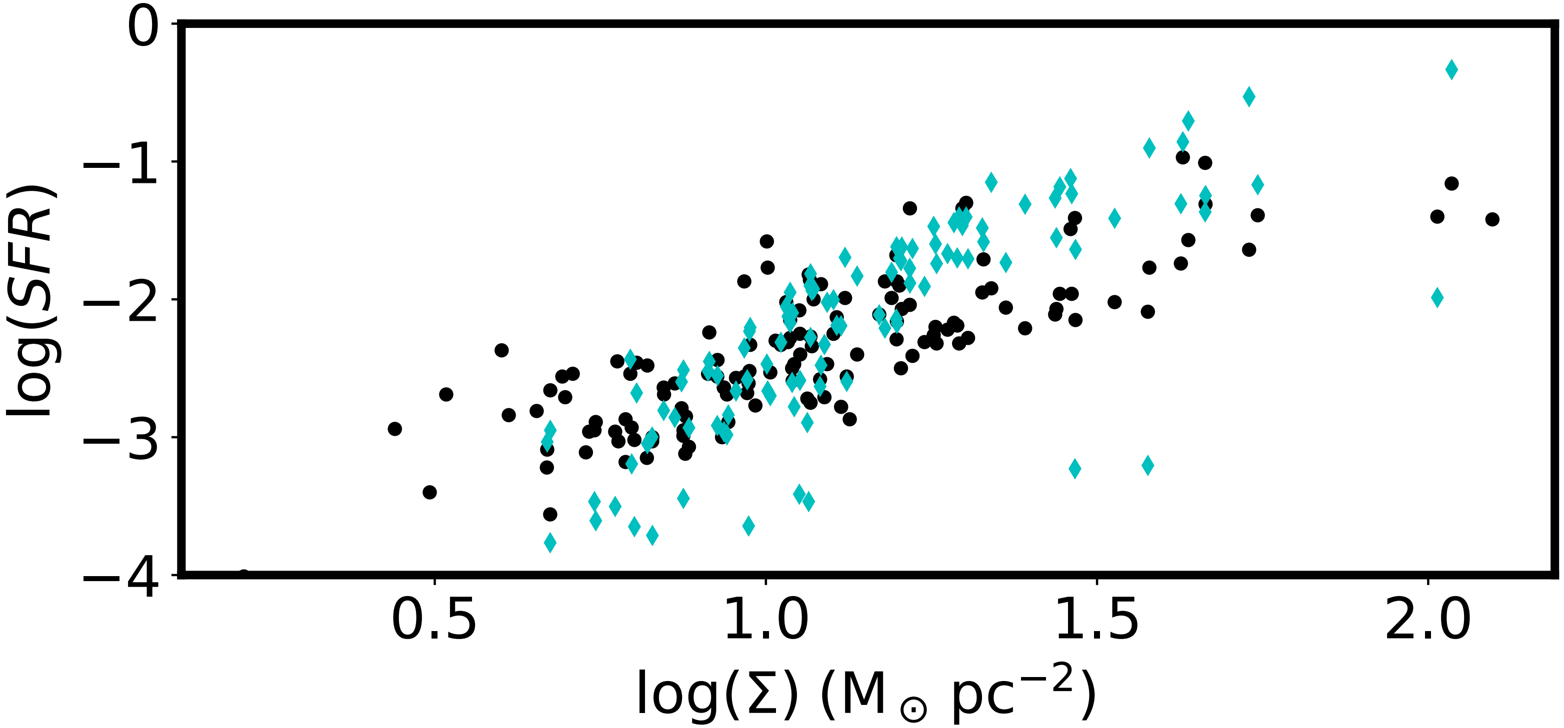


Comparison with a model that is *good*

For each data point (De Los Reyes & Kennicutt 2019), we have:

- -gas column density
- -stellar column density
- -rotation

MHD model , narrow log-Poissonl density PDF, Q_*=1.2



Conclusions

-Galactic box simulations seem to require external driving and/or strong magnetization
Stellar feedback is not enough

-Classical SFR models suffer strong inconsistencies and fail to reproduce high Mach simulations

-Turbulent support model seems to be doing a reasonable job but several properties such as density pdf, density powerspectrum are needed

-the SK relation is likely a consequence of :

Large scale turbulence+magnetic field+stellar feedback

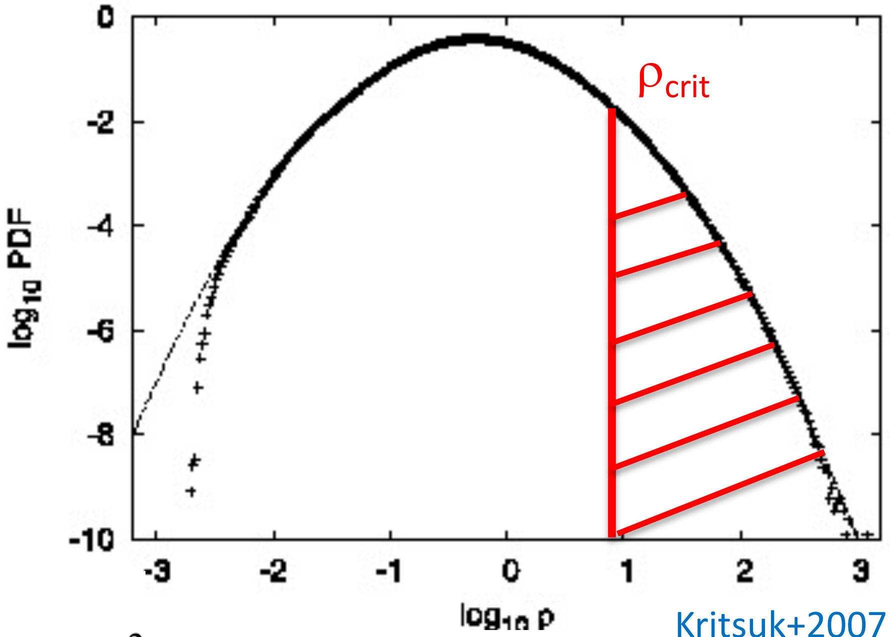
Classical analytical models for the Star formation rate

Krumholz&McKee 2005, Padoan&Nordlund 2011, H&Chabrier 2011, 2013, Renaud+2012

Log-normal PDF : turbulence and no gravity

$$\mathcal{P}(\delta) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\delta-\bar{\delta})^2}{2\sigma_0^2}\right), \quad \delta = \ln(\rho/\rho_0)$$

$$\bar{\delta} = -\sigma_0^2/2\,, \quad \sigma_0^2 = \ln(1 + b^2\mathcal{M}^2),$$



Kritsuk+2007

Critical density from sonic length

$$\lambda_s = R_0 \left(\frac{c_s}{\sigma_0}\right)^2 = R_0 \mathcal{M}^{-2}, \qquad \rho_{\rm crit,KM} = \pi \frac{c_s^2 \mathcal{M}^4}{G R_0^2} = \frac{4}{5}\pi\rho_0 \alpha_{\rm vir} \mathcal{M}^2$$

Summing-up over the PDF weighted by mass and freefall

$$\mathrm{SFR}_{ff}^{\rm simp} = \epsilon \int_{\delta_{\rm crit}}^{\infty} \frac{\tau_{ff}^0}{\tau_{ff}(\rho)\phi_t} \widetilde{\rho}\mathcal{P}(\delta)d\delta = \frac{\epsilon}{\phi_t} \int_{\delta_{\rm crit}}^{\infty} \widetilde{\rho}^{3/2}\mathcal{P}(\delta)d\delta$$

$$= \frac{\epsilon}{2\phi_t} \exp\left(3\sigma_0^2/8\right)\left[1 + \mathrm{erf}\left(\frac{\sigma_0^2 - \ln(\widetilde{\rho}_{\rm crit})}{2^{1/2}\sigma_0}\right)\right]. \quad (8)$$

H&Chabrier 2011

Why a freefall time rather than a replenishement time?

The density PDF over which we integrate, is set by turbulence
The question is then over which timescale dense gas is being replenished
There is no reason that the dense gas is replenished in a local freefall time

Why a density threshold?

A piece of fluid can collapse at any density if big enough

Why no turbulent support?

Turbulence can disperse a piece of fluid if strong enough

Why spatial distribution of mass not accounted for?

Flows with very different powerspectra can have same PDF
Flows broken in small entities may be stable against gravity

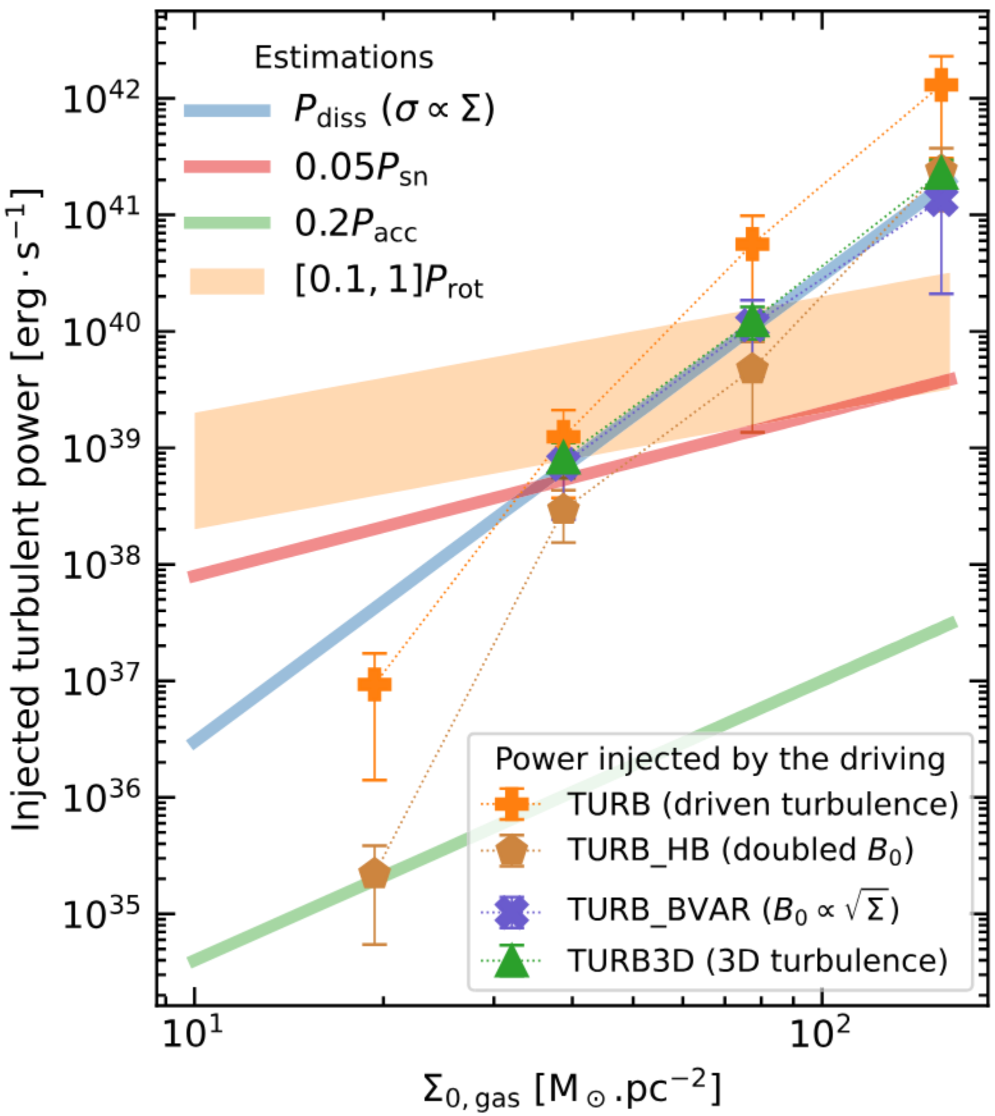
An estimate of the replenishment time
« PDF » weigthed turbulent scale dependent crossing time

$$\tau_{\rm cross}(R') = \frac{R'}{\sigma(R')}. \qquad \sigma(R') = \sigma_0 \left(\frac{R'}{L_i}\right)^{\eta_v} \qquad R'/R = \rho/\rho'.$$

$$\tau_{R,R'} = \frac{\tau_{\rm cross}(R')}{\epsilon(R,R')} = \frac{L_i^{\eta_v}}{\epsilon(R,R')\sigma_0} R^{1-\eta_v} \left(\frac{\rho}{\rho'}\right)^{1-\eta_v}$$

$$\begin{aligned}\tau_{\rm rep}(R) &= \frac{\int_{-\infty}^{\ln(\rho)} \tau_{R,R'} \rho' \mathcal{P}(\rho')\,{\rm d}\ln(\rho')}{\int_{-\infty}^{\ln(\rho)} \rho' \mathcal{P}(\rho')\,{\rm d}\ln(\rho')}, \\ &= \frac{L_i^{\eta_v}}{\sigma_0} R^{1-\eta_v} \rho^{1-\eta_v} \frac{\int_{-\infty}^{\ln(\rho)} \epsilon(R,R')^{-1} (\rho')^{\eta_v} \mathcal{P}(\rho')\,{\rm d}\ln(\rho')}{\int_{-\infty}^{\ln(\rho)} \rho' \mathcal{P}(\rho')\,{\rm d}\ln(\rho')},\end{aligned}$$

Which source of energy to feed the turbulence?



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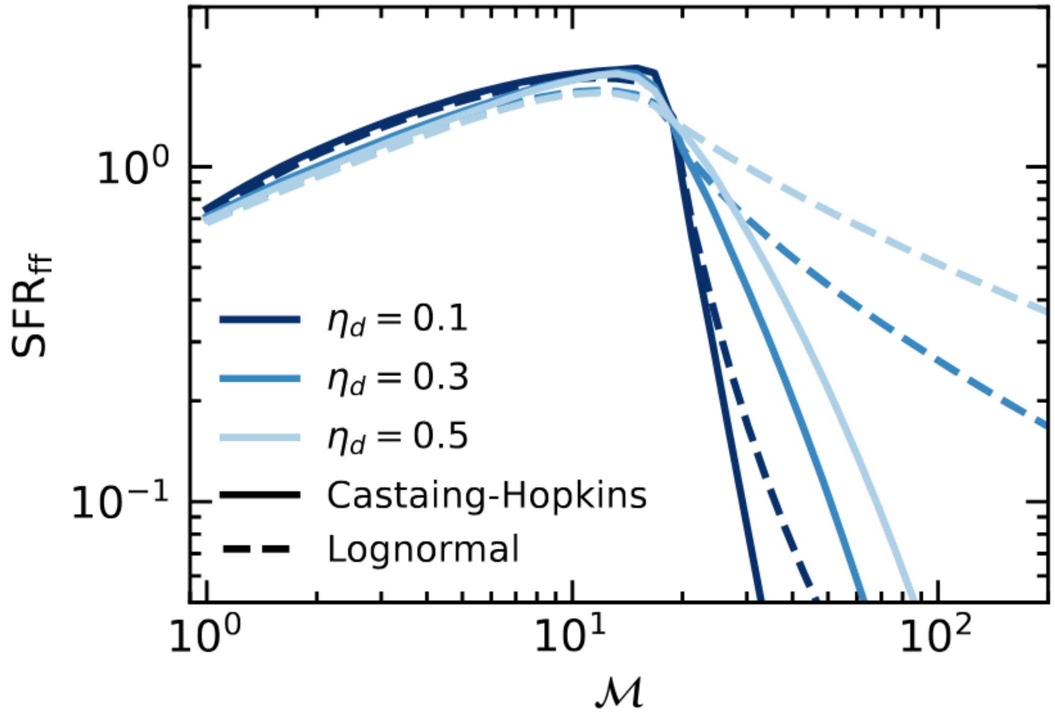
Predicted SFR as a function of Mach number for different log ρ powerspectra

Most important feature: at high Mach, the SFR drops steeply.



6 cases corresponding to

-2 different PDF
-3 different powerspectrum of log ρ

This happens when:
the turbulent injection length / the size of the system is comparable to the *turbulent Jeans length*

=> No available gravitationally unstable density fluctuations

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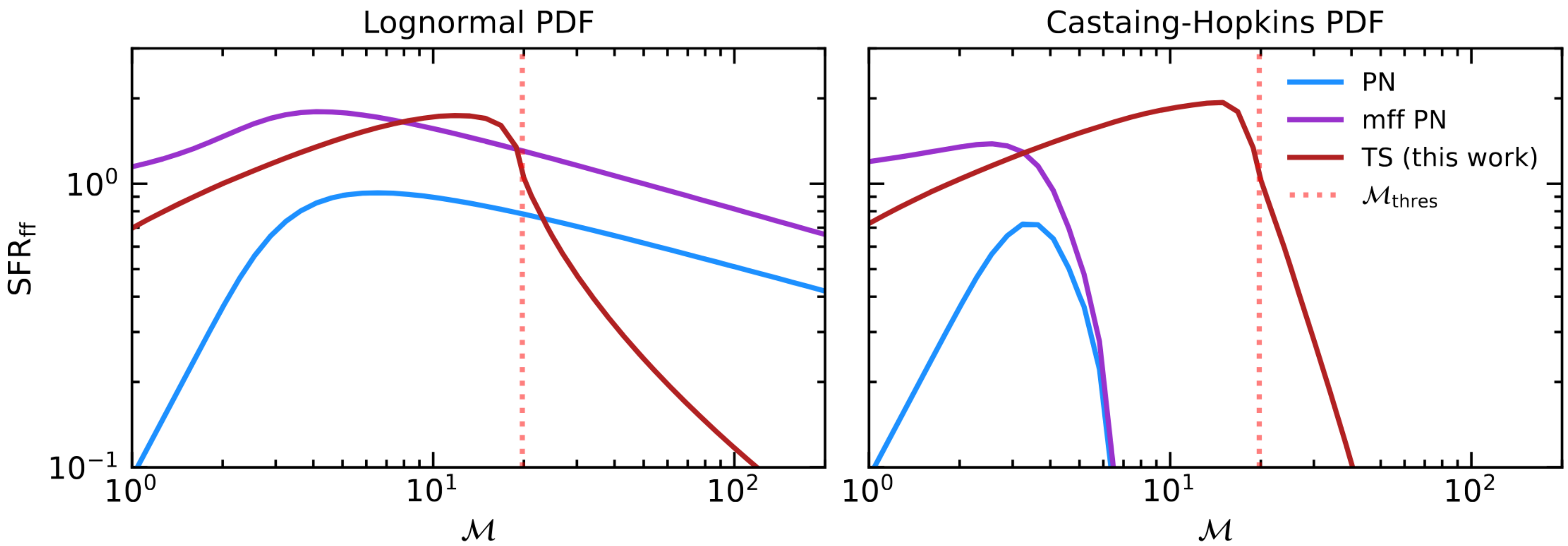
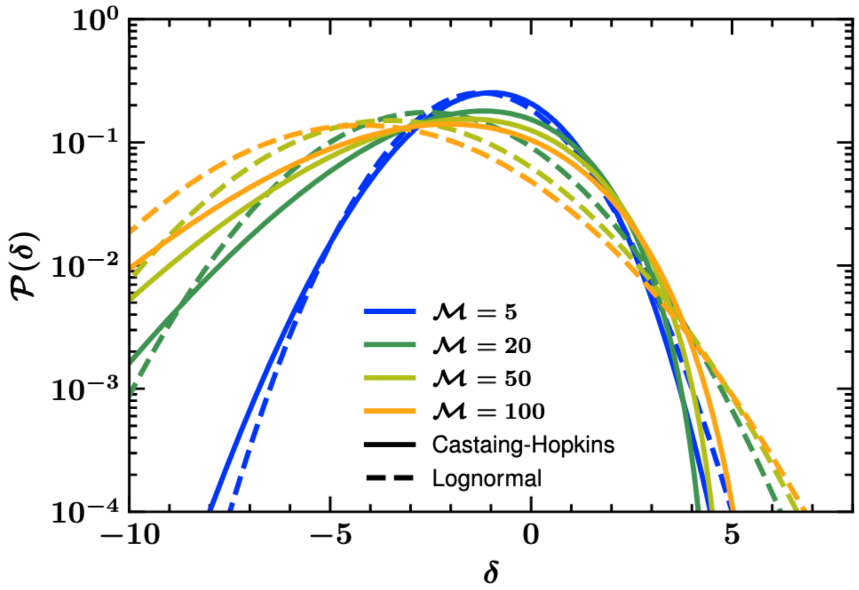
Comparison between classical models and turbulent support one for 2 density PDF





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