MCMC orbital fitting using Universal Keplerian Variables

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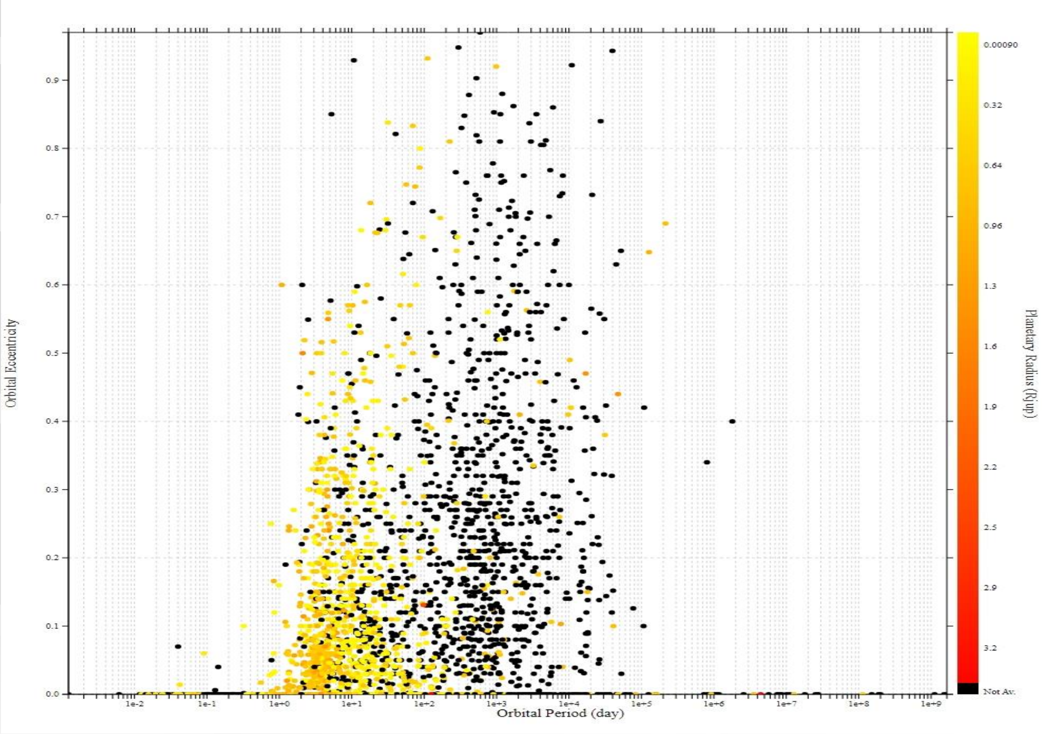
Exoplanets

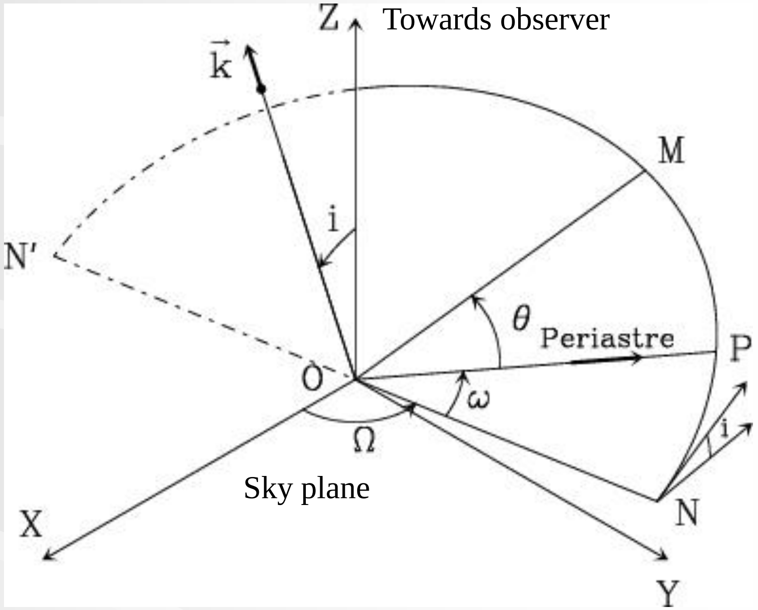
- More than 6400 exoplanets are known today, including more than 4800 planetary systems and 1000 multiple planet systems.
- Orbital architectures of these system are incredibly diverse : large/small eccentricities, resonant systems, compact systems, large distance planets...
- Orbital reconstruction, based on available observations, is an important issue to investigate dynamical stability, history and relation with environment of these systems.



exoplanet.eu

Orbital reconstruction

- How do you recover 3D information about the orbits of exoplanets from available, sparse observational data ?
- What information ? Dynamical masses + orbital parameters *$(a, e, i, \Omega, \omega, t_p)$ + mass or Period*
- 4 possible types of data:
  - Relative astrometry (HCI) : $[x-x_*, y-y_*] = f(t)$
  - Stellar Radial velocities (Spectroscopy) : $v_{z*} = f(t)$
  - Planet radial velocities (Planet spectrum): $v_z - v_{z*} = f(t)$
  - Absolute stellar astrometry (Gaia) : $[x_*, y_*] = f(t)$
  - + transits, TTVs, etc…
  - We sometimes have several types of data (combined fit), but not always…

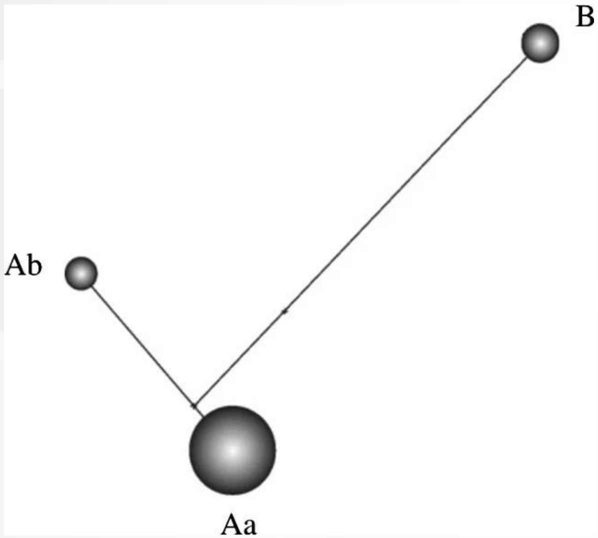


- This is today viewed as a Bayesian inference issue : Given
  - the sparseness and precision of the available data, bad orbital coverage,
  - general or external priors/constraints on parameters,

⇒ what is the posterior distribution of orbital elements ?
⇒ MCMC !

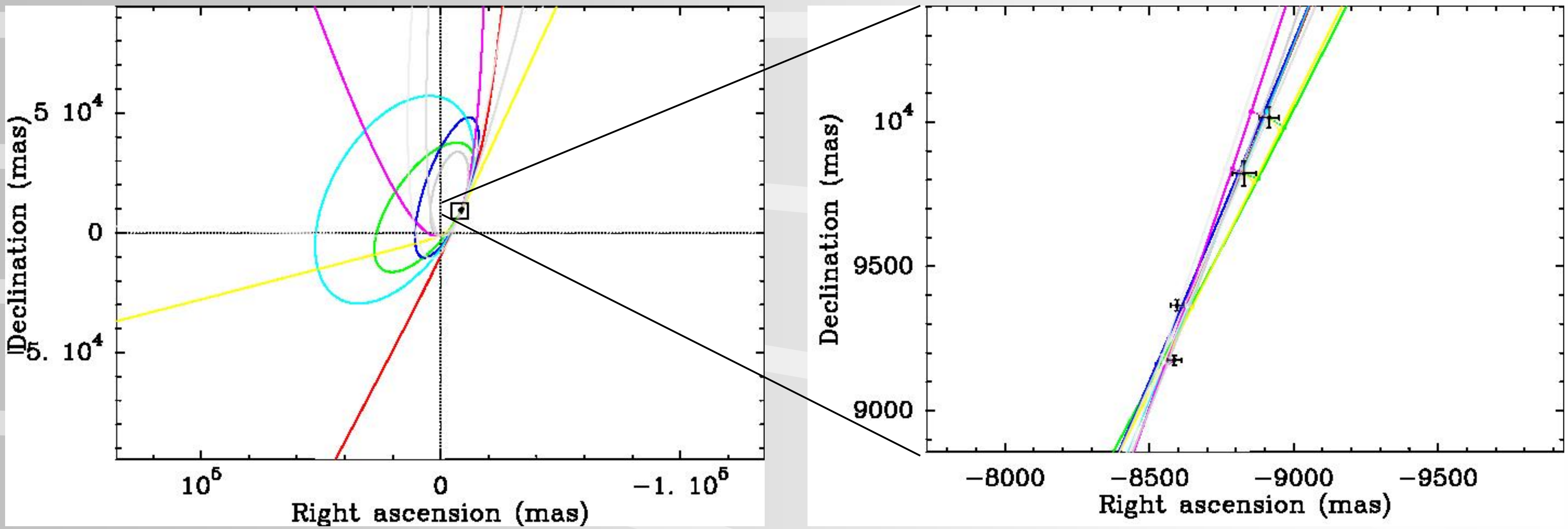
Multiple planets / components systems

- This tends to get more complex when several planets are detected in a single system
- True motions are not strictly Keplerian ⇒ Need for a convenient definition of planetary coordinate to fit Keplerian orbits
- Helio(stello)-centric coordinates : Each planet is referred to its position with respect to the central star (independently from others) : $\boldsymbol{r} = \boldsymbol{r}_b - \boldsymbol{r}_*$
  - ⇒ Relevant when the central mass is really dominant, but problematic in the case of inner planet
- Jacobi coordinates : Each planet (or companion) is referred to its position with respect to the center of mass of bodies below it : $\boldsymbol{r}_i = \boldsymbol{r}_{b,i} - cm(\boldsymbol{r}_*, \boldsymbol{r}_{1,b}, .. \boldsymbol{r}_{i-1,b})$
  - ⇒ More accurate description of hierarchical systems with nested orbits
  - ⇒ but more complicate expression of mutual distances and interactions
- In any case : More planets ⇒ larger parameter space ⇒ more complex problem.



Problems with eccentric and poorly sampled orbits

- Long period planets (HCI) : Only few positions measured, often on one side of the orbit



- In some extreme cases, what is measured is no more than a projected position and a projected velocity.
- Vertical coordinate and velocity unknown $\Rightarrow$ no theoretical upper limit to the eccentricity : high eccentricity, even unbound solutions may exist !
- Semi-major axis distribution with extended tail $\Rightarrow$ convergence problems

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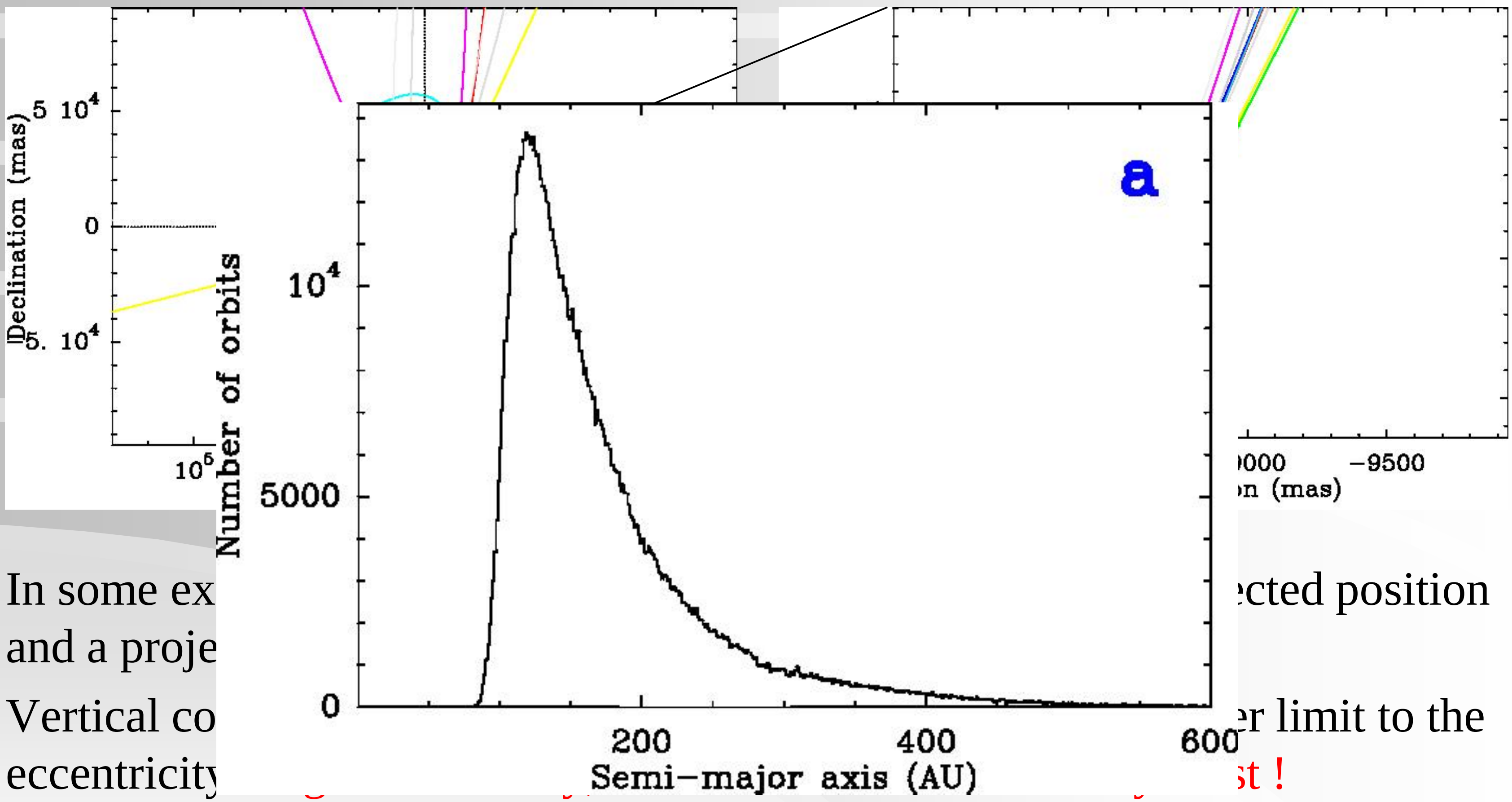

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Classical / universal variables description

- Classical formulation ⇔ Try to fit orbital elements (or combinations of them) using classical Keplerian formulas : Link time ↔ position (M=*mean anomaly* ≈ time : u=*eccentric anomaly* ≈ position)
- Problem : Formulas are different depending on whether the orbit is bound or unbound ⇒ instabilities / convergence issues for MCMC for $e \sim 1$

$$u - e\sin u = M \qquad \frac{u}{2} + \frac{u^3}{6} = M \qquad e\sinh u - e = M$$

(elliptic) (parabolic) (hyperbolic)

- ⇒ Need for a unique, continuous formulation ⇒ Universal Keplerian Variables (Danby & Burkardt 1893, Burkardt & Danby 1983, Danby 1987, Beust et al. 2016)

$$\alpha = \frac{GM}{q}(1-e), \quad s = \frac{u}{\sqrt{|\alpha|}} = \frac{1-e}{q}\left(t-t_p\right) + \frac{eY}{\sqrt{qGM(1+e)}}$$

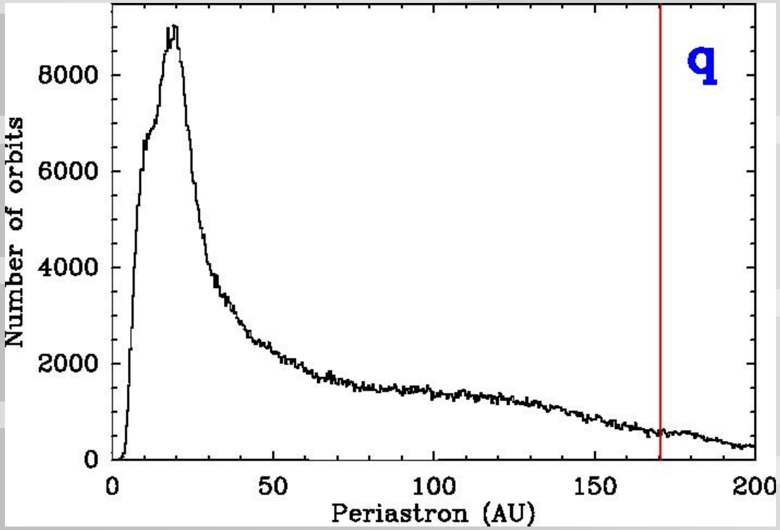
$$\text{Kepler:} \quad GM\, s^3 c_3\left(\alpha s^2\right) + qsc_1\left(\alpha s^2\right) = t - t_p$$

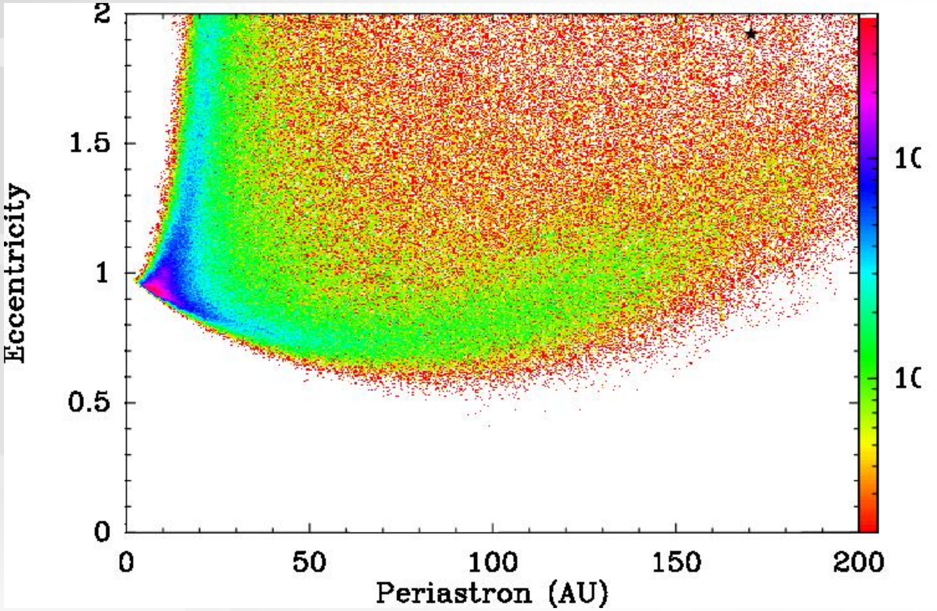
$$c_k(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+k)!} x^n \qquad \text{(Stumpff functions)}$$

Our code (Oracle)

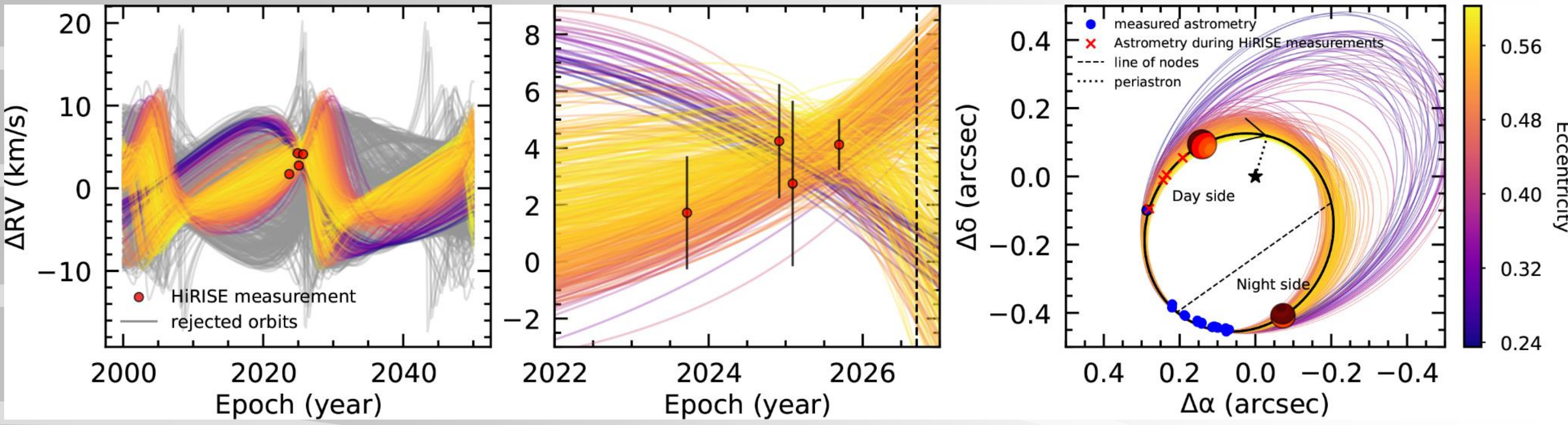
- MCMC code dedicated to fit orbits of imaged exoplanets, allowing combining various kinds of data.
- Allows to simultaneously fit several orbits is a given system, making use of Jacobi coordinates.
- Main code in Fortran 90 with Python package for solutions handling and generating various plots, and generating input files.
- Dedicated to be included into the DIVA+ database (part of HC_DC SNO) for general use.
- Parallelized with OPEN-MP (parallel treatment of walkers)
- Makes use of Universal or Classical Keplerian Variables (optional)
- Allows to add external priors on individual and combinations on masses.
- Proved to achieve better convergence than orbitize! (Denis et al. 2026)

Application example 1 : Fomalhaut b



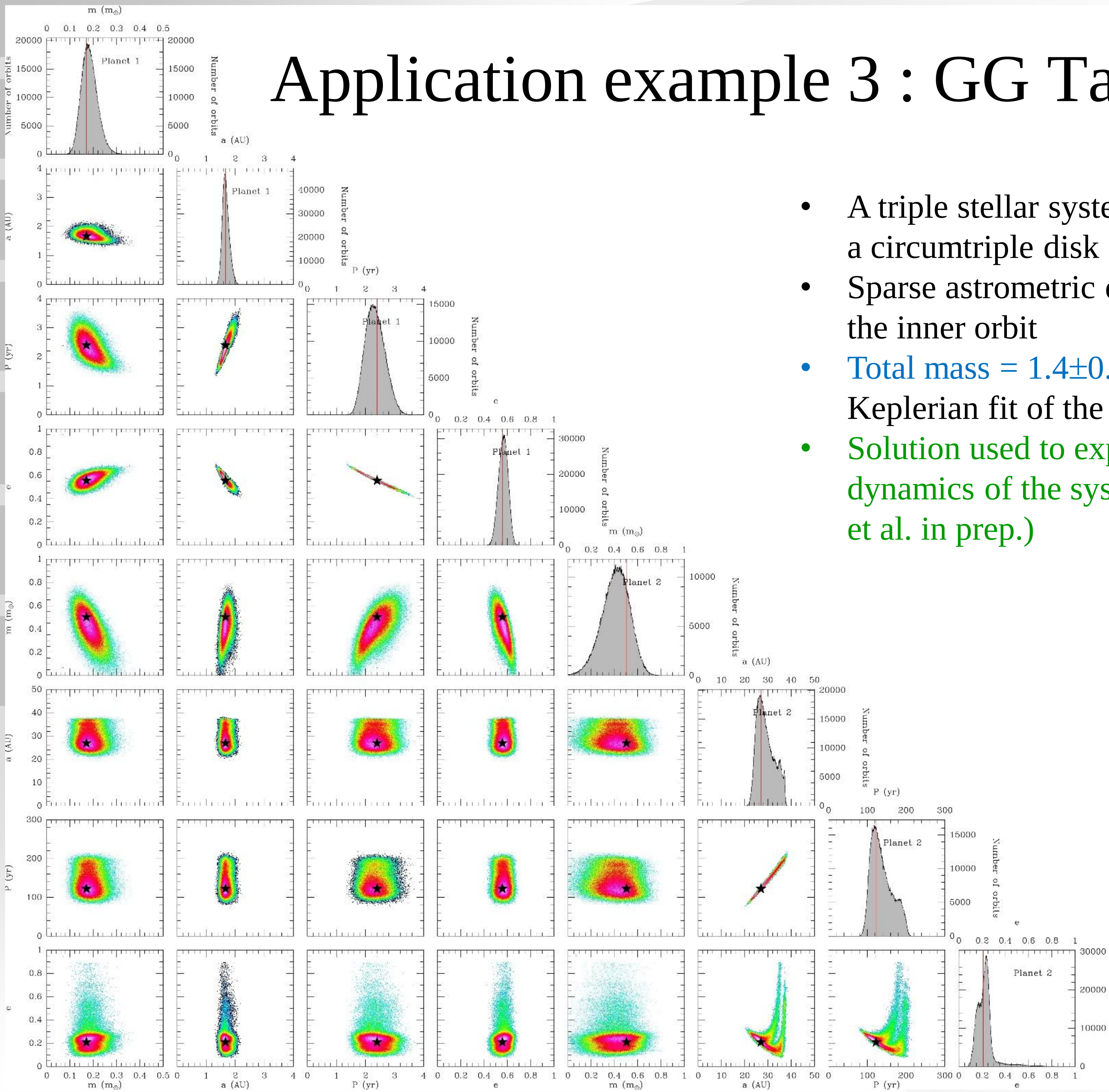


Application example 2 : 51 Eri b



Denis et al., 2026, A&A 707, L13

Application example 3 : GG Tau A



- A triple stellar system surrounded by a circumtriple disk
- Sparse astrometric data, especially for the inner orbit
- Total mass = 1.4±0.05 $M_{\odot}$, thanks to Keplerian fit of the disk ⇒ prior
- Solution used to explore the overall dynamics of the system (Lacquement et al. in prep.)