Artificial intelligence in radiative hydrodynamics for astrophysics

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What are shocks?

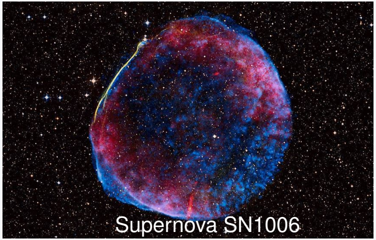
Propagating disturbance that moves faster than the local speed of sound in the medium.

Hydrodynamic shock

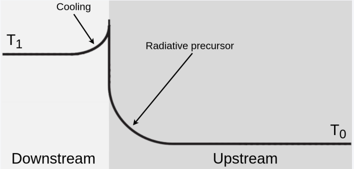
Credit: *Realbigtaco / Wikimedia Commons*

Shock structure

Radiative shock



Credit: *NASA, ESA, Zolt Levay (STScI)*



Shock structure

Model to describe a radiative shock

Radiative hydrodynamics model

Hydrodynamics equations:

$$\begin{cases} \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) & = 0 \\ \partial_t (\rho \vec{v}) + \vec{\nabla} \cdot (\rho \vec{v} \otimes \vec{v} + p) & = \vec{S} \\ \partial_t \mathrm{E} + \vec{\nabla} \cdot ((\mathrm{E} + p) \vec{v}) & = cS^0 \end{cases}$$

- Density ρ,
- Momentum $\rho \vec{v}$,
- Energy E,
- Pressure p

Perfect gas closure relation:

$$\mathrm{p} = (\gamma - 1) \left\{ \mathrm{E} - \rho v^2/2 \right\}$$

M1-gray equations:

$$\begin{cases} \partial_t \mathrm{E}_R + \vec{\nabla} \cdot \vec{F}_R & = -cS^0 \\ \partial_t (c^{-2} \vec{F}_R) + \vec{\nabla} \cdot \mathbb{P}_R & = -\vec{S} \end{cases}$$

- Radiative energy E_R,
- Radiative flux $\vec{F}_R$,
- Radiative pressure $\mathbb{P}_R$

M1 closure relation:

$$\mathbb{P}_R = \mathbb{D}_R \mathrm{E}_R$$

$\mathbb{D}_R$: Eddington tensor

(Levermore 1984 ; Dubroca & Feugeas 1999 ; Radureau et al. 2025)

Model to describe a radiative shock

Radiative hydrodynamics model

Hydrodynamics equations:

$$\begin{cases} \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \\ \partial_t (\\ \partial_t E \end{cases}$$

- Dens
- Mome
- Energ
- Pressu

M1-gray equations:

$$\partial_t E_R + \vec{\nabla} \cdot \vec{F}_R = -cS^0$$

HADES-2D code

2D radiative hydrodynamics code using a Cartesian mesh, used in the study of astrophysical phenomena.

Michaut et al. (2011), Nguyen (2011), Michaut et al. (2017)

Perfect gas closure relation:

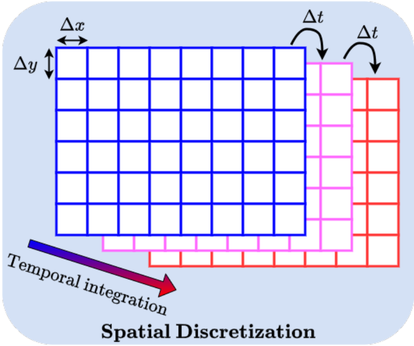
$$p = (\gamma - 1)\left\{E - \rho v^2/2\right\}$$

M1 closure relation:

$$\mathbb{P}_R = \mathbb{D}_R E_R$$

$\mathbb{D}_R$: Eddington tensor
(Levermore 1984 ; Dubroca & Feugeas 1999 ; Radureau et al. 2025)

Resolution method



- Δx, Δy are fixed
- Δt is determined by the *CFL condition*

Courant–Friedrichs–Lewy (CFL) stability condition

Hydrodynamics

$$\Delta t_h \leq \Delta x / v$$

Radiation hydrodynamics

$$\Delta t_r \leq \Delta x / c$$

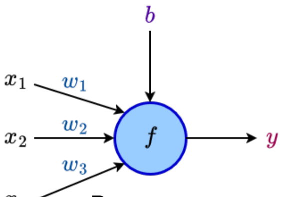
Comparison of both time steps

$$v \approx 10^4 \text{ m/s}, \quad c \approx 10^8 \text{ m/s}$$

$$\Rightarrow \quad \Delta t_r \approx 10^{-4} \, \Delta t_h$$

Very high computational cost

Neural Networks



Perceptron

Base structure: the Perceptron

Input/output relationship:

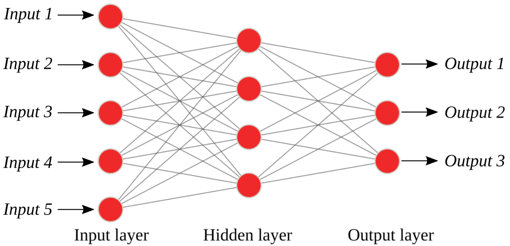
$$y = f\left(\sum_i w_i x_i + b\right)$$

Where:

- w_i : weights,
- b : Biases,
- f : Activation function.

Multi-Layer Perceptron

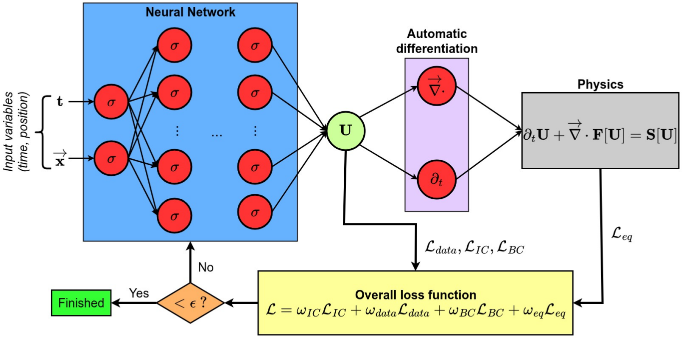
Assembly of perceptrons in layers.



Multi-Layer Perceptron

Universal function approximator
(Hornik et al. 1989)

Physics-Informed Neural Networks



Error terms:

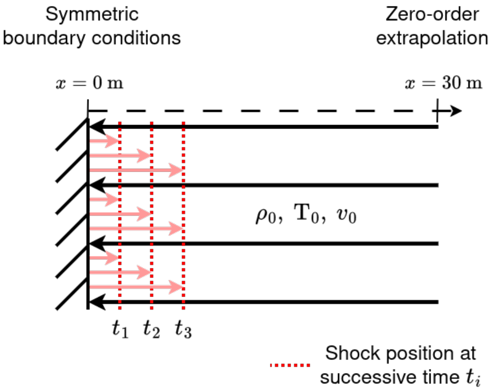
- $\mathcal{L}_{data}$: simulation data;
- $\mathcal{L}_{IC}$: initial conditions;
- $\mathcal{L}_{BC}$: boundary conditions;
- $\mathcal{L}_{eq}$: equations.

Residual in $\mathcal{L}_{eq}$ $\quad$ $\mathbf{res} = \lambda \left\{ \partial_t \mathbf{U} + \vec{\nabla} \cdot \mathbf{F}[\mathbf{U}] - \mathbf{S}[\mathbf{U}] \right\}^2$

Weight λ (Liu et al., 2023) $\quad$ $\lambda = \left\{ 1 + \epsilon \left(|\vec{\nabla} \cdot \vec{v}| - \vec{\nabla} \cdot \vec{v} \right) \right\}^{-1}$

Physics equations incorporated in the error

Physical test configuration



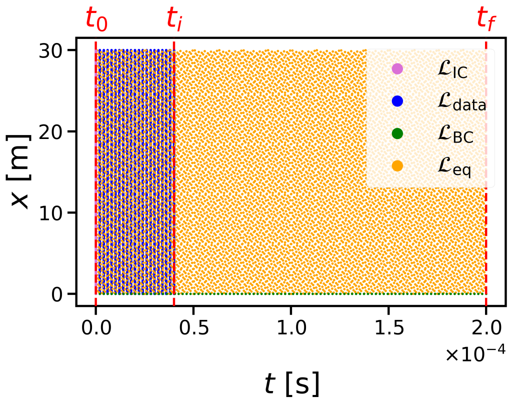
Initial Conditions

- Density : $\rho_0 = 10^{-3}$ g/cm^3
- Temperature : $T_0 = 4.3$ eV
- Velocity : $v_0 = 50$ km/s

Tested Configurations

- **Hydrodynamic shock**
  Mach number: $M_{hydro} = 16$
- **Radiative shock**
  Constant mean free path: $\ell = 1$ m
  Mach number: $M_{rad} = 11$

Data repartition for the different error terms

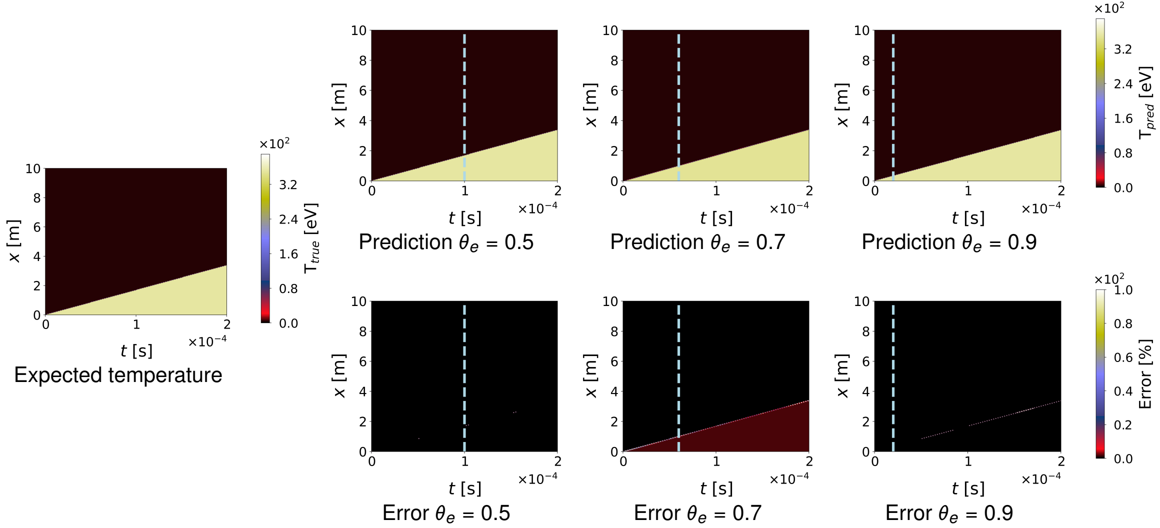


Extrapolation Degree

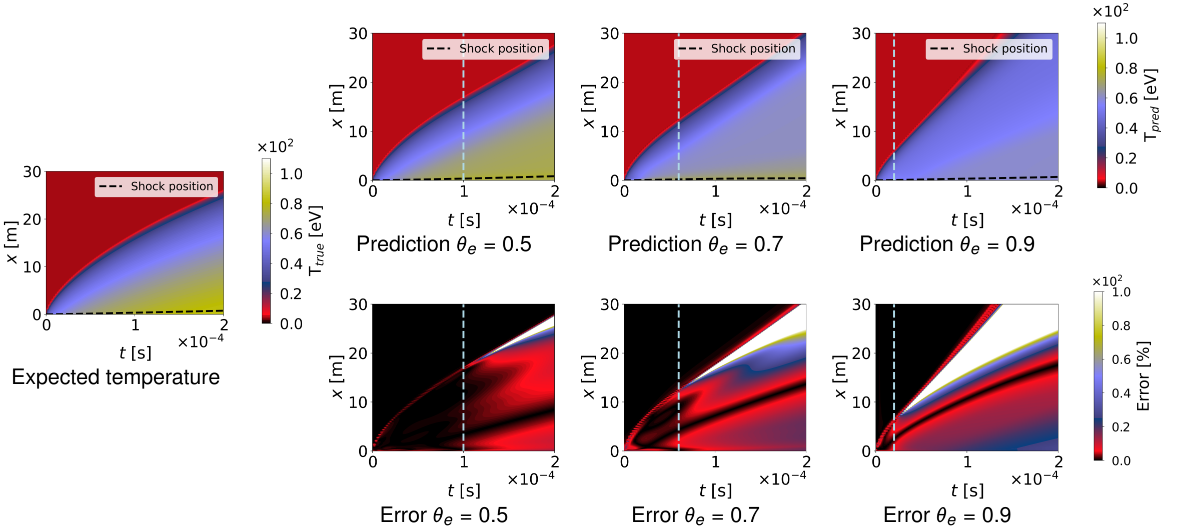
$$\theta_e = \frac{t_f - t_i}{t_f - t_0}$$

- $\theta_e \to 0$ No extrapolation
- $\theta_e \to 1$ Far extrapolation

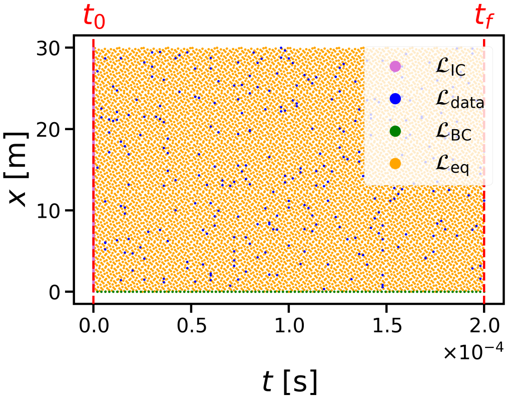
Hydrodynamic shock - extrapolation



Radiative shock - extrapolation



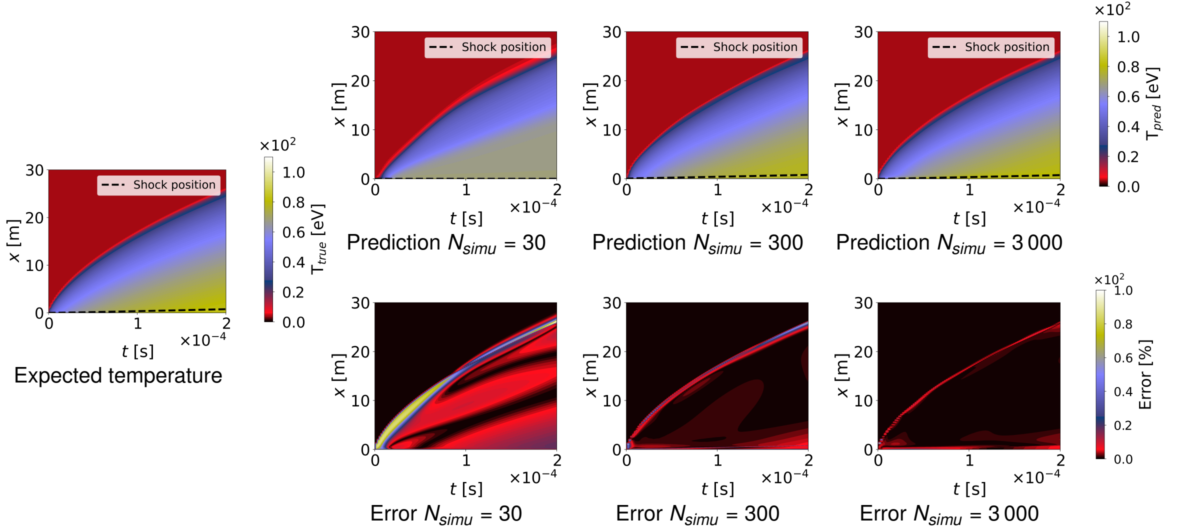
Radiative shock - interpolation: data repartition



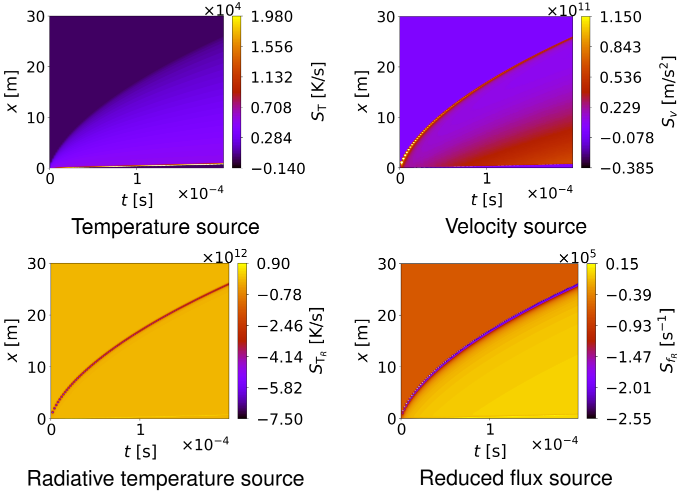
Parameter of interest:

Number of simulation data N_{simu}.

Radiative shock - interpolation



Source of the problem in radiative shocks (Radureau et al. under review)



Governing equations used

Hydrodynamics	*Radiation*
$\partial_t \rho = \dots$	$\partial_t T_R = \cdots + S_{T_R}$
$\partial_t T = \cdots + S_T$	$\partial_t f_R = \cdots + S_{f_R}$
$\partial_t v = \cdots + S_v$	

Stiff source terms appear at the edge of the radiative precursor

Radiative temperature : $T_R = \left(\frac{E_R}{a_R}\right)^{1/4}$

Reduced flux : $f_R = \frac{F_R}{cE_R} \in]-1, 1[$

Conclusions and Perspectives

Conclusions on PINNs

- Hydrodynamic shocks — accurate long-term extrapolation;
- Radiative shocks — failure due to discontinuity + stiff source terms;
- Radiative shocks — Successful interpolation using sparse data.

Future work

- **PINNs**: investigate formulations better suited for discontinuities and stiff source terms (e.g., **weak-form PINNs**, **adaptive** / **domain-decomposition methods**);
- Beyond **PINNs**: explore alternative AI approaches for radiative shocks (e.g., **Neural Operators**)

Thank you for your attention!

References:

Radureau et al., PINNs for radiative shocks (accepted)

Radureau et al., Phys. Rev. E 2025 — radiative shocks with the M1-multigroup model

Radureau et al., Phys. Rev. E 2025 — M1 multigroup Eddington factor

Acknowledgements:

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Loss function - Equations

Radiative hydrodynamics equations:

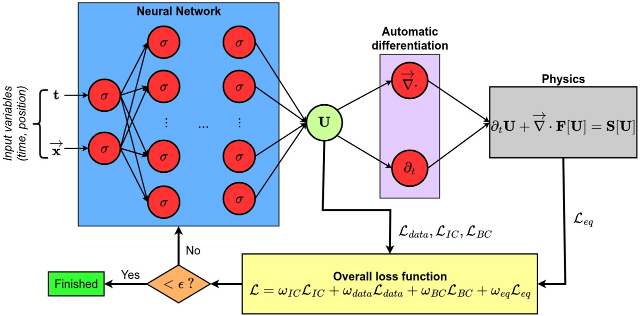
$$\begin{cases} \partial_t \rho + v\partial_x \rho + \rho\partial_x v & = 0 \\ \partial_t v + v\partial_x v + \mathcal{R}\left(\mathrm{T}\frac{\partial_x \rho}{\rho} + \partial_x \mathrm{T}\right) & = S_v \\ \partial_t \mathrm{T} + v\partial_x \mathrm{T} + (\gamma - 1)\mathrm{T}\partial_x v & = S_\mathrm{T} \\ \partial_t \mathrm{T}_R + c\left(\mathrm{f}_R \partial_x \mathrm{T}_R + \frac{\mathrm{T}_R}{4}\partial_x \mathrm{f}_R\right) & = S_{\mathrm{T}_R} \\ \partial_t \mathrm{f}_R + c\left(\left(\chi_R' - \mathrm{f}_R\right)\partial_x \mathrm{f}_R + 4\left(\chi_R - \mathrm{f}_R^2\right)\frac{\partial_x \mathrm{T}_R}{\mathrm{T}_R}\right) & = S_{\mathrm{f}_R} \end{cases} \Rightarrow \partial_t \mathbf{U} + \mathbb{A}\partial_x \mathbf{U} = \mathbf{S}[\mathbf{U}]$$

where:

$$S_v = \frac{a_R \mathrm{T}_R^4}{\rho \ell}\left\{\mathrm{f}_R - \beta\left[\chi_R + \left(\mathrm{T}/\mathrm{T}_R\right)^4\right]\right\}, \quad S_\mathrm{T} = \frac{c\mathrm{T}}{\ell \mathrm{R}}\left\{1 - \left(\mathrm{T}/\mathrm{T}_R\right)^4 - 2\beta \mathrm{f}_R\right\},$$

$$S_{\mathrm{T}_R} = -\frac{c\mathrm{T}_R}{4\ell}\left\{1 - \left(\mathrm{T}/\mathrm{T}_R\right)^4 - \beta \mathrm{f}_R\right\}, \quad S_{\mathrm{f}_R} = -\frac{c}{\ell}\left\{\mathrm{f}_R\left(\mathrm{T}/\mathrm{T}_R\right)^4 - \beta\left[\chi_R - \mathrm{f}_R^2 + \left(\mathrm{T}/\mathrm{T}_R\right)^4\right]\right\},$$

Loss function - Equations



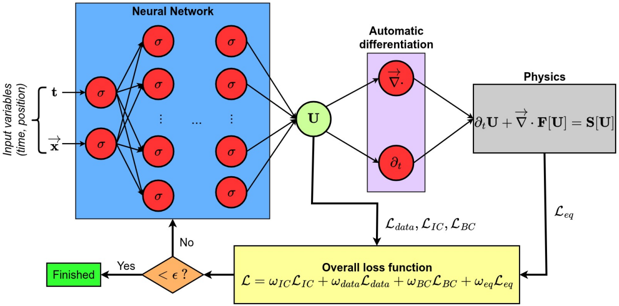
Equation loss function:

$$\mathcal{L}_{eq} = \frac{1}{N_{eq}} \sum_{i=1}^{N_{eq}} \lambda_i ||\mathbf{R}(t_i, x_i)||^2 \quad .$$

Residuals:

$$\mathbf{R}(t_i, x_i) = \partial_t \mathbf{U}(t_i, x_i) + \mathbb{A}(t_i, x_i)\partial_x \mathbf{U}(t_i, x_i) - \mathbf{S}[\mathbf{U}(t_i, x_i)]$$

Loss function - Initial conditions



Initial condition loss function:

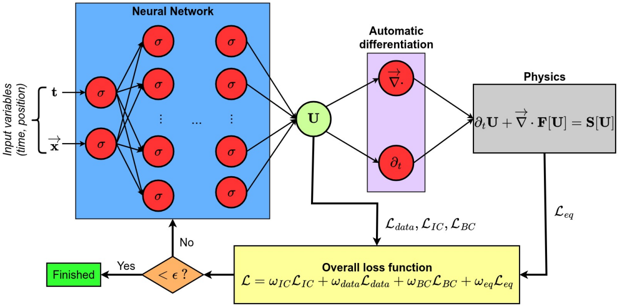
$$\mathcal{L}_{IC} = \frac{1}{N_{IC}} \sum_{i=1}^{N_{IC}} \left|\left|\mathbf{I}[\mathbf{U}(t_0, x_i)] - \mathbf{I}[\mathbf{U}_0(x_i)]\right|\right|^2 ,$$

where:

$$\mathbf{I}[\mathbf{U}] = \begin{bmatrix} \log(\rho) \\ \text{asinh}(v) \\ \log(\mathrm{T}) \\ \log(\mathrm{T}_R) \\ \text{asinh}(\alpha_{\mathcal{F}} \mathrm{f}_R) \end{bmatrix} ,$$

and $\alpha_{\mathcal{F}} = 10^5$.

Loss function - Boundary conditions



Boundary condition loss function:

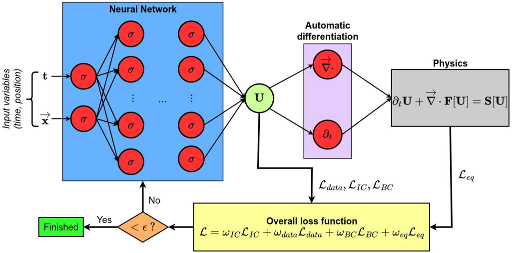
$$\mathcal{L}_{BC} = \frac{1}{N_{BC}} \sum_{i=1}^{N_{BC}} ||\mathbf{B}[\mathbf{U}(t_i, x_{min})]||^2 \ ,$$

where:

$$\mathbf{B}[\mathbf{U}] = \begin{bmatrix} \text{asinh}(v) \\ \text{asinh}(\alpha_{\mathcal{F}} f_R) \end{bmatrix} \ ,$$

and $\alpha_{\mathcal{F}} = 10^5$.

Loss function - Simulation data



Simulation data loss:

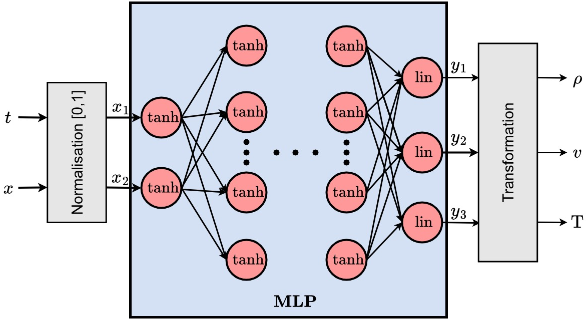
$$\mathcal{L}_{data} = \frac{1}{N_{simu}} \sum_{i=1}^{N_{simu}} ||\mathbf{I}[\mathbf{U}(t_i, x_i)] - \mathbf{I}[\mathbf{U}_{simu}(t_i, x_i)]||^2 ,$$

where:

$$\mathbf{I}[\mathbf{U}] = \begin{bmatrix} \log(\rho) \\ \mathrm{asinh}(v) \\ \log(\mathrm{T}) \\ \log(\mathrm{T}_R) \\ \mathrm{asinh}(\alpha_{\mathcal{F}} \mathrm{f}_R) \end{bmatrix} ,$$

and $\alpha_{\mathcal{F}} = 10^5$.

Neural network - hydrodynamics

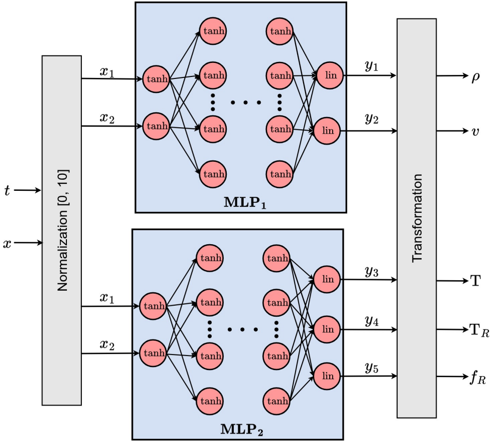


MLP
Number of hidden layer : 3
Number of neuron per hidden layer: 1

Transformation:

$$\rho = |w_\rho|\ \mathrm{SP}(y_1) + |b_\rho|$$
$$v = |w_v|\ y_2 + b_v$$
$$\mathrm{T} = |w_T|\ \mathrm{SP}(y_3) + |b_T|$$

Neural network - radiative hydrodynamics



MLP$_1$
Number of hidden layer : 2
Number of neuron per hidden layer: 20

MLP$_2$
Number of hidden layer : 2
Number of neuron per hidden layer: 20

Transformation:

$$\rho = |w_\rho|\ \mathrm{SP}(y_1) + |b_\rho|$$
$$v = |w_v|\ y_2 + b_v$$
$$\mathrm{T} = |w_T|\ \mathrm{SP}(y_3) + |b_T|$$
$$\mathrm{T}_R = |w_T|\ \mathrm{SP}(y_4) + |b_T|$$
$$\mathrm{f}_R = \frac{|w_{\mathrm{f}_R}|\ y_5}{\alpha_{\mathcal{F}}}$$

and $\alpha_{\mathcal{F}} = 10^5$.