

Orion B, ESA/Herschel/PACS, SPIRE/N. Schneider,
Ph. André, V. Könyves

Neural network estimation of the average density in molecular clouds and prestellar cores : A benchmark of modern architectures

Zack Ribeiro & Pierre Hily-Blant

Captured by Frédéric CHIOLA

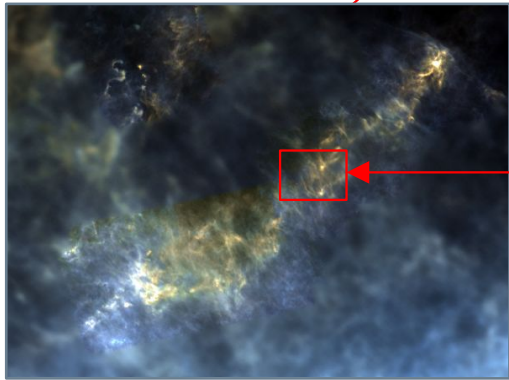


Dense cores and star formation

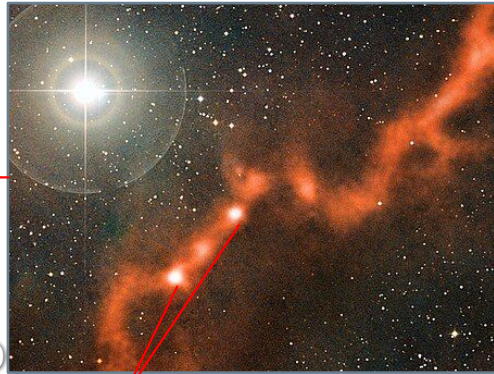
- Dense cores are the birth sites of stars.
- Critical parameters :
 - Density profil, radius, mass
- Critical questions :
 - Star Formation Rate ; Origin of the initial mass function

Observational challenge

- Dust observations provide column density
- Star formation depends on volume density
- Recovering volume density is a degenerate problem



(B, Hasenberger et al. 2018), Infrared emission

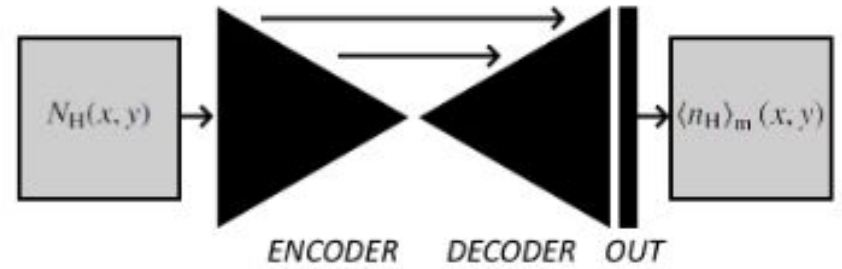
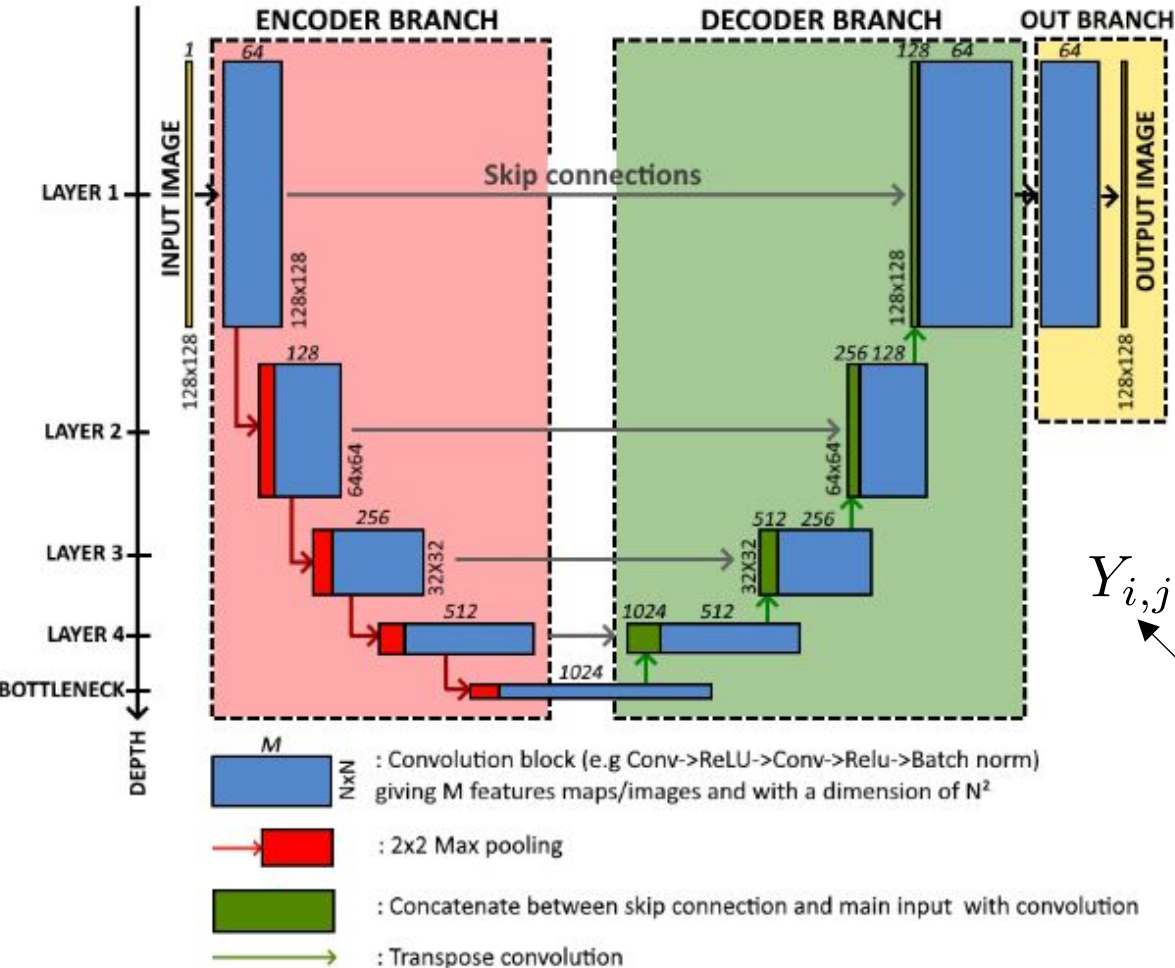


Credit : ESO/APEX A. Hacar et al.

Dense cores

$$N_{\text{H}}(x, y) = \int_0^L n_{\text{H}}(x, y, z) dz. \rightarrow \langle n_{\text{H}} \rangle_{\text{m}}(x, y) = \frac{1}{N_{\text{H}}} \int_0^L n_{\text{H}}^2(x, y, z) dz.$$

Unet - Convolutional Neural Networks



(Unet, Ronneberger et al (2015) ; Oktay et al (2018) ; LeCun et al (1989))

$$Y_{i,j} = f\left(\sum_{m=1}^M \sum_{n=1}^N W_{m,n} \cdot X_{i+m,j+n} + b\right)$$

Pixel at (i,j) in the resulting image Y

Element (m,n) of the kernel

Width and Height of the used kernel (M=N=3)

Conditional Invertible Neural Networks (cINNs)

- **Generate** samples by a sequence of invertible transformations :

$$Z = f_K \cdot f_{K-1} \cdot \dots \cdot f_1(X, c).$$
 From sample from Gaussian prior, to a data sample conditioned.

$$p_z = \tilde{N}(0, I) \longrightarrow x_0 \sim P_{\text{data}}.$$

- **Training** : Learn to transform the complex data distribution into a gaussian one using **invertible** transformations.

$$p_{\text{data}}(X|c) = p_z(f(X, c)) \left| \det \frac{\partial f(X, c)}{\partial X} \right|.$$

- **Inference** : Network is **inverted** in order to transform a sample from the gaussian distribution to the learned data distribution

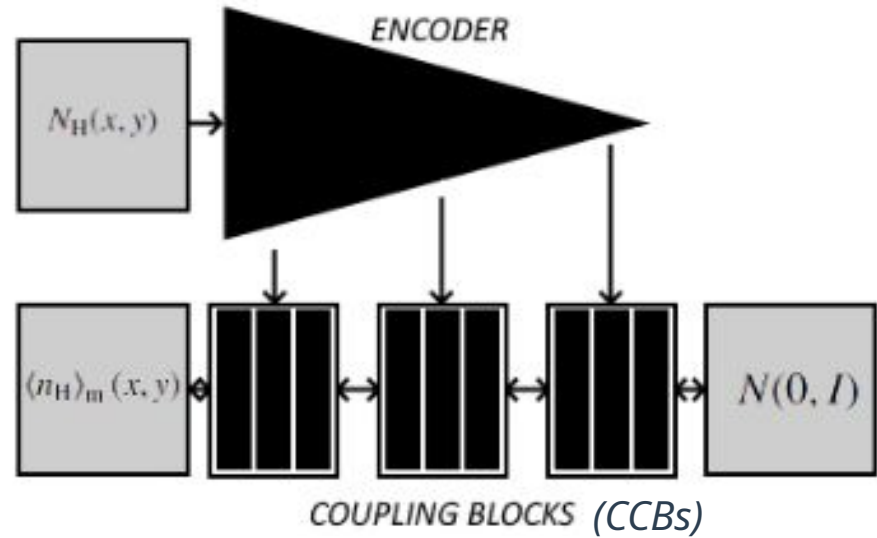
$$X = f^{-1}(Z|c).$$

transformation of a CCB is:

$$\begin{cases} v_1 = u_1 \odot \exp(s_1(u_2, c)) + t_1(u_2, c), \\ v_2 = u_2 \odot \exp(s_2(v_1, c)) + t_2(v_1, c), \end{cases} \quad \begin{cases} u_2 = (v_2 - t_2(v_1)) \odot \exp(-s_2(v_1)) \\ u_1 = (v_1 - t_1(u_2)) \odot \exp(-s_1(u_2)) \end{cases}$$

where s_i, t_i are scale and translation functions ←

Learned by convolutional neural networks (CNNs)



(cINNs, Ardizzone et al. (2021) ; Rezende & Mohamed (2016))

✦ Denoising Diffusion Probabilistic Models (DDPMs)

- **Generate** samples by gradually transforming white noise, i.e. sample from Gaussian prior, into a data sample.

$$p_z = \bar{N}(0, I) \longrightarrow x_0 \sim p_{\text{data}}$$

- **Training** : a U-Net learn to predict the noise added to $\langle n_H \rangle_m$

$$dx_t = f(x, t)dt + g(t)dW_t = -\frac{1}{2}\beta(t)x_t dt + \sqrt{\beta(t)}dW_t,$$

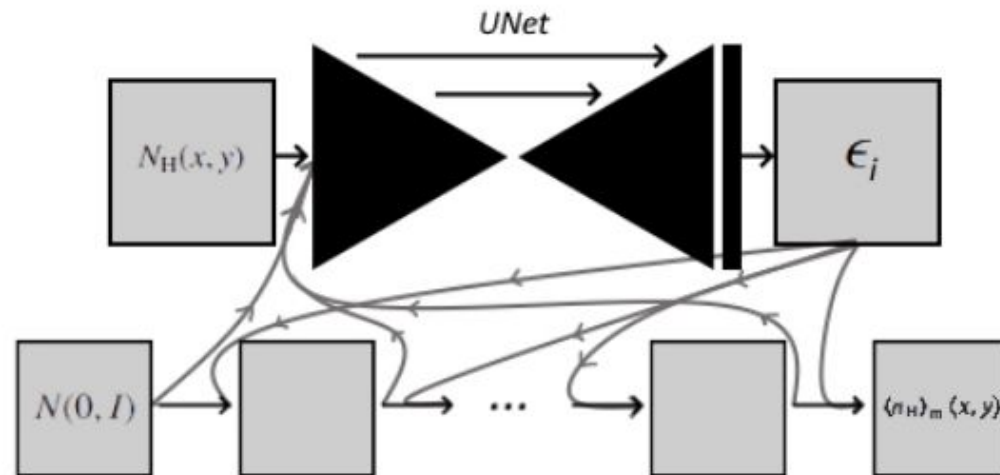
where W_t is a Wiener process. Here:

- $g(t) = \sqrt{\beta(t)}$ is the diffusion strength,
- $f(x, t) = -\frac{1}{2}\beta(t)x_t$ is the drift term,
- $t \in [0, T]$ parametrized the diffusion trajectory.

- **Inference** : Sequence of learned denoising steps

$$dx_t = [f(x, t) - g(t)^2 \nabla_{x_t} \log p_t(x_t)]dt + g(t)d\bar{W}_t,$$

Likelihood

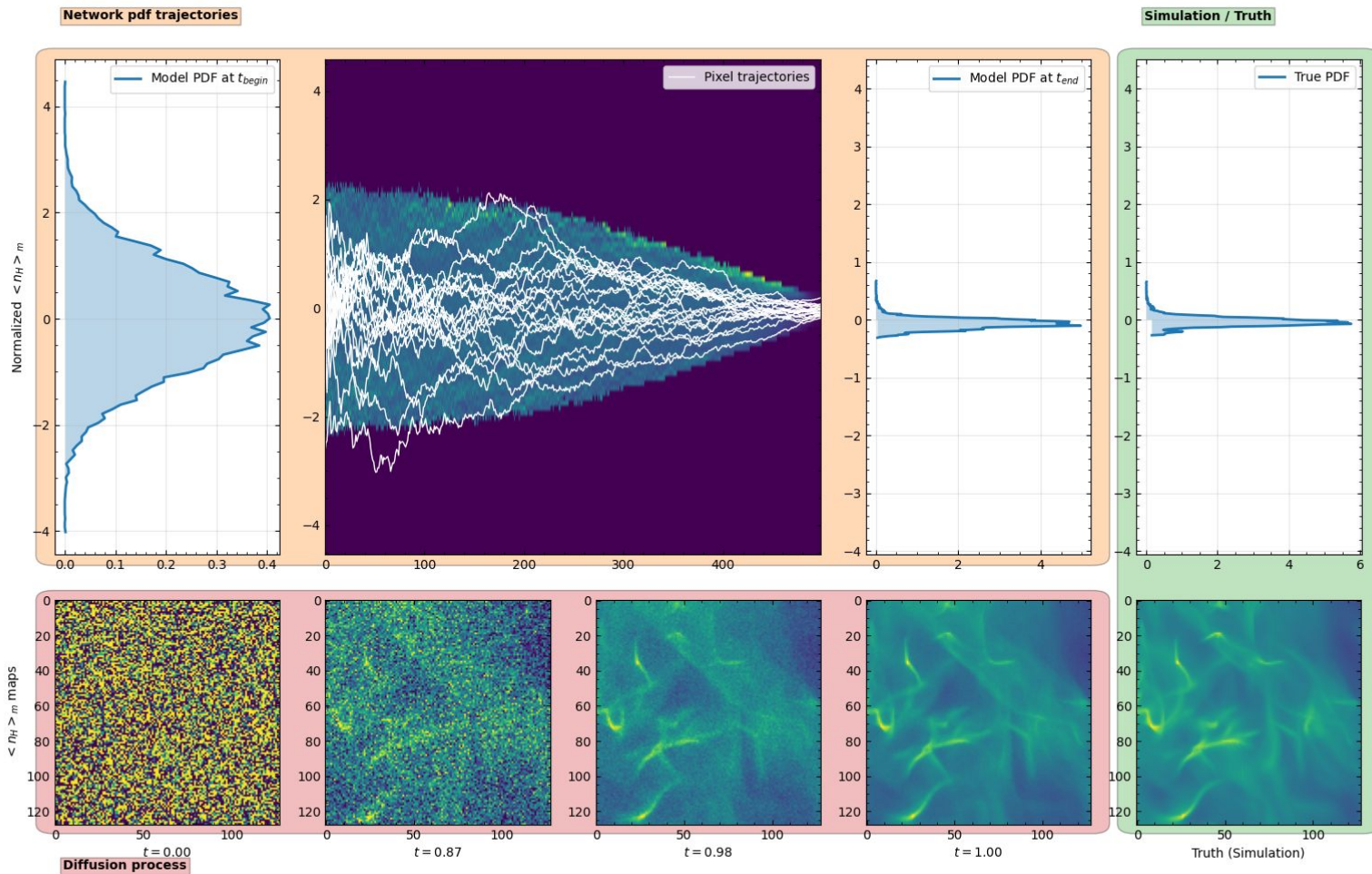


REVERSE/DENOISING PROCESS

(DDPMs, Ho et al. (2020))

DDPMs – Diffusion trajectories

(DDPMs, Ho et al. (2020))



- Each element of the map follows a **diffusion process**
- Each **amplitude** of diffusion step is **predicted by a U-Net**.

Dataset & Training

Simulation (RAMSES)

Ntormousi & Hennebelle (2019, GALACTICA database)

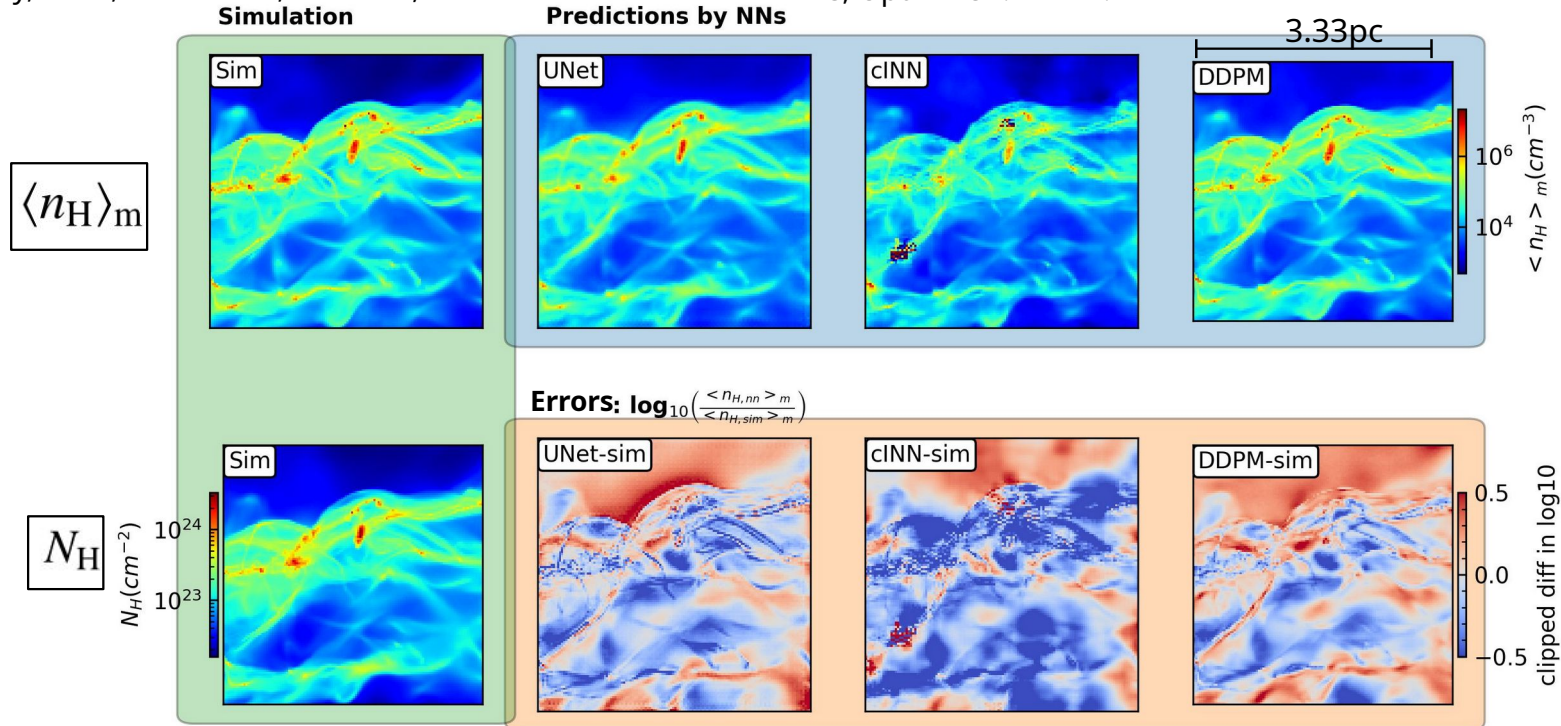
- Width: 66,1pc, resolution : AMR (down to 500au)
- Turbulence, Gravity, Magnetic field (Ideal MHD), Not isothermal

+ Simulation (IDEFIX) for validation only

- Turbulence, Gravity, MHD, Isothermal, Periodic ; res = 512

Training : 100 regions

- 128px - 3.33pc wide
- 70 % for training, 30 % for validation.
- Diffuse to dense regions
- Data augmentation, ExpMovAverage (EMA) for cINNs and DDPMs, Optimizer : ADAM.



Benchmark on validation-set (from Simulations)

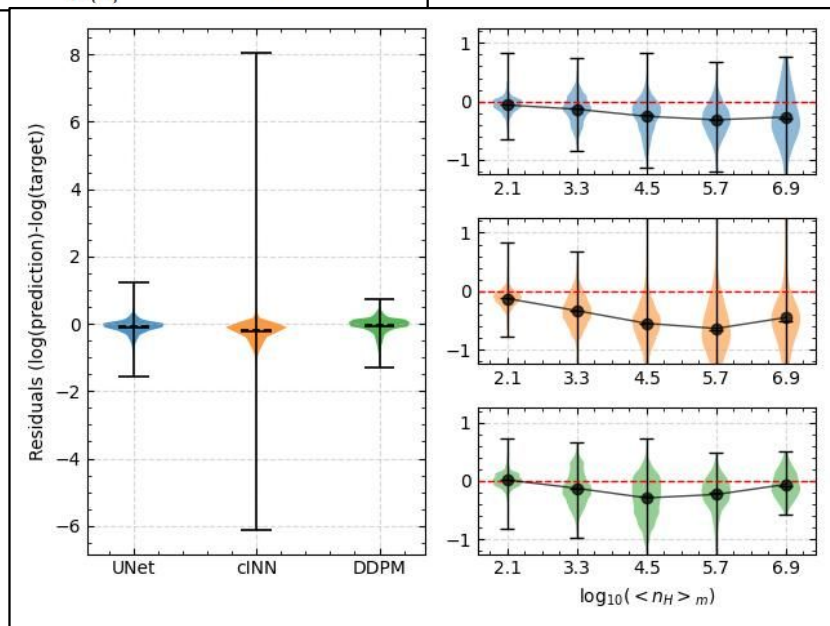
- **U-Net:** faster inference and training, great performance (**factor 1.25**)
- **DDPMs:** slower but same performance and probabilist, i.e can handle degenerate problems.
- **cINNs:** unstable and difficult to train but probabilist with exact likelihood

| Metric | UNet | cINN | DDPM |
|------------------------------|------|------|------|
| MSE (log10) | 0.20 | 0.50 | 0.25 |
| Parameters ($\times 10^7$) | 13 | 3.8 | 0.8 |
| Inference speed (regions/s) | 3.5 | 3.4 | 0.32 |
| Error at 80% accuracy | 0.25 | 0.39 | 0.25 |
| Training stability | High | Low | High |

(GPU : RTX 2070 SUPER)

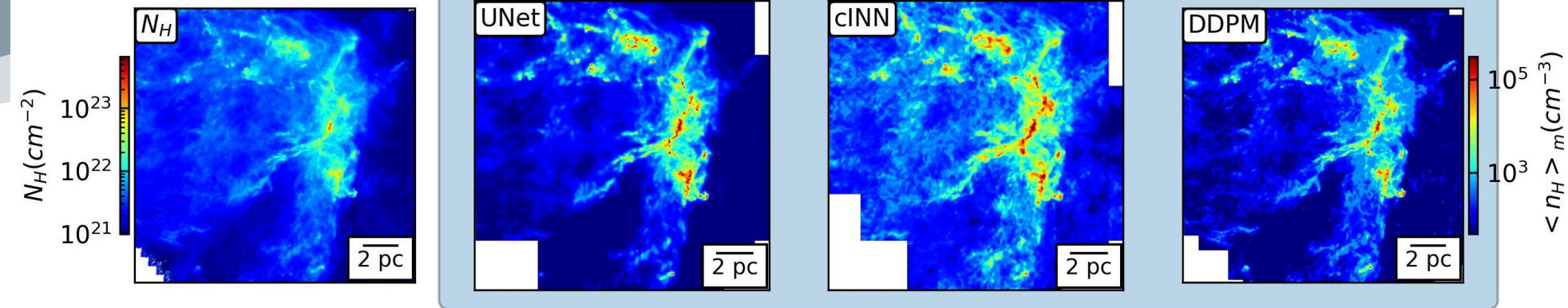
$$\text{Acc}(\sigma) = \frac{1}{B} \sum_{i \in \{B\}} \frac{N(|Y_{\text{pred}} - Y_{\text{true}}| < \sigma)}{N_i}$$

$$\text{MSE} = \mathbb{E} \left[\left(\log Y_{\text{pred}} - \log Y_{\text{true}} \right)^2 \right],$$

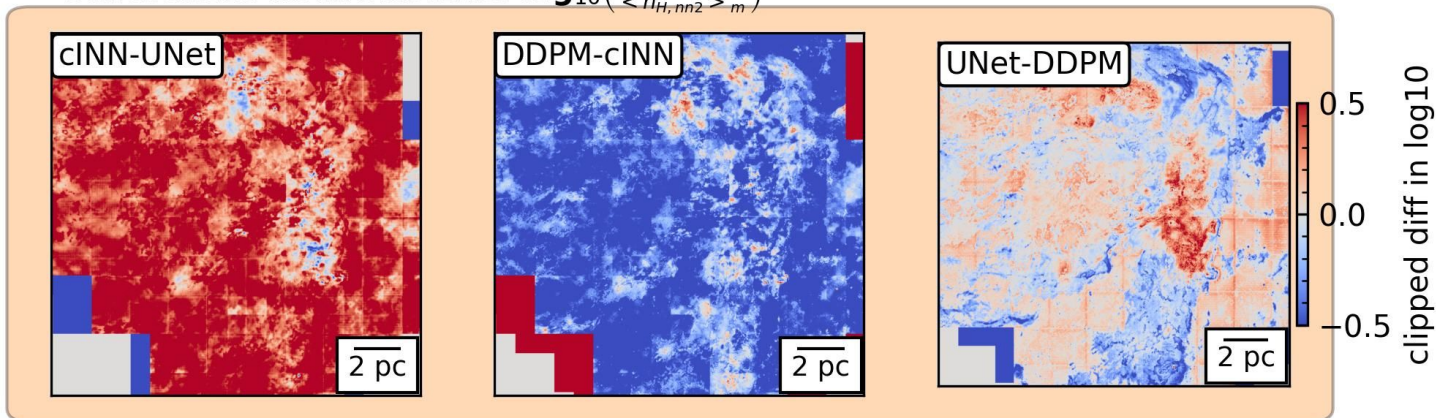


Application on Orion B

Predictions: $\langle n_H \rangle_m$



Differences between NNs: $\log_{10} \left(\frac{\langle n_{H,nn1} \rangle_m}{\langle n_{H,nn2} \rangle_m} \right)$



(in prep, Ribeiro & Hily-Blant (2026))

- **Core catalog** from *Könyves et al. (2020)*.

$$\frac{\langle n_H \rangle_m}{n_c} = \frac{n_d}{n_c} + \frac{n_c L_c}{N_H} \left(1 - \frac{n_d}{n_c} \right)$$

- **cINNs** : best for predicting dense cores densities ;
DDPMs, U-Net : for the diffuse environment

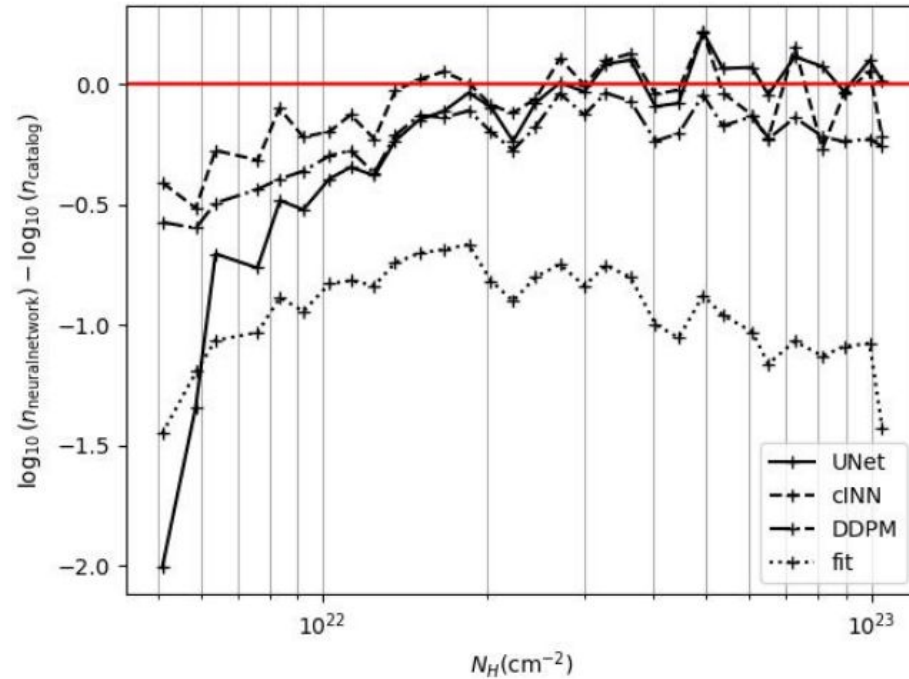
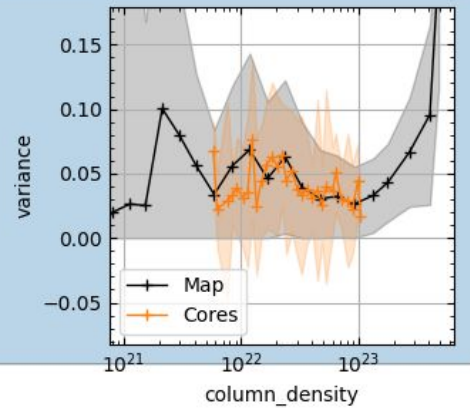
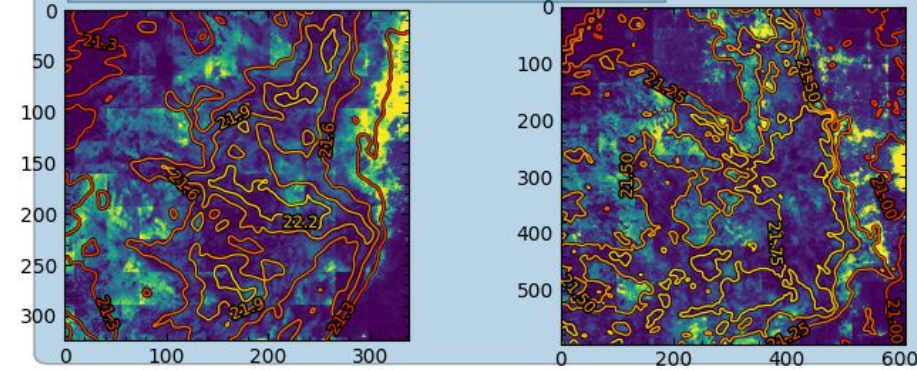


Fig. 10. Log-binned averages of differences in logspace between predicted average core densities and catalogued values. (≈ 400 cores)

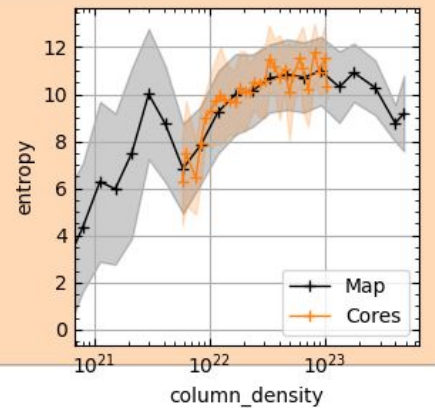
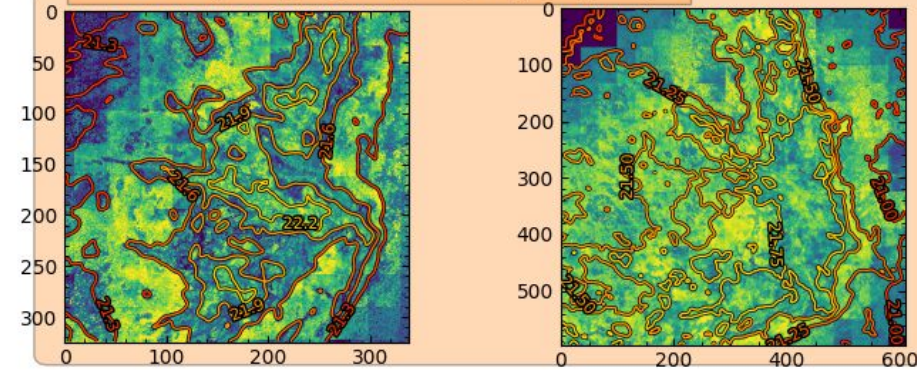
Advantages of using probabilistic models

$$\text{VARIANCE: } \sum ((p_{(n), \text{pred}}(\log_{10}(n)) - \sum p_{(n), \text{pred}} \times (n)) / \sigma_{n, \text{val}})^2$$



- **Probabilistic models predict distributions.**
→ Possible to :
 - **measure model error**
 - **difficulty** of predicting certain regions.

$$\text{ENTROPY: } H_{x,y} = \exp(-\sum p_{<n>, \text{pred}}(x,y) \log(p_{<n>, \text{pred}}(x,y)))$$



- **Degeneracy is not uniform** across the cloud and column density.

Region 1

Region 2

Results

- **Neural networks can estimate physical properties** (as the volume density) in molecular clouds by extracting and combining information from observational maps.
- **Probabilistic** neural networks can address **degenerate problems**.

Limitations

- **Dust emission only is limited**, an architecture capable of also using molecular emission to leverage some degeneracy (e.g separation of structures along the line of sight) is needed.
- **Missing simulations** to achieve peak performance of cINN and to validate using external simulations.

Future

- **New target : Catalog of cores**, multi-modal using molecular emissions and dust emission. → New measurement of CMF by taking into account the multiplicity along l.o.s

Benchmark on Orion B

- **Core catalog** from *Könyves et al. (2020)*.
- Radii and core list are **unchanged**; densities are **replaced** with our estimates.

$$\frac{\langle n_H \rangle_m}{n_c} = \frac{n_d}{n_c} + \frac{n_c L_c}{N_H} \left(1 - \frac{n_d}{n_c} \right)$$

- cINNs : best for predicting dense cores densities ; DDPMs, U-net : for the diffuse environment

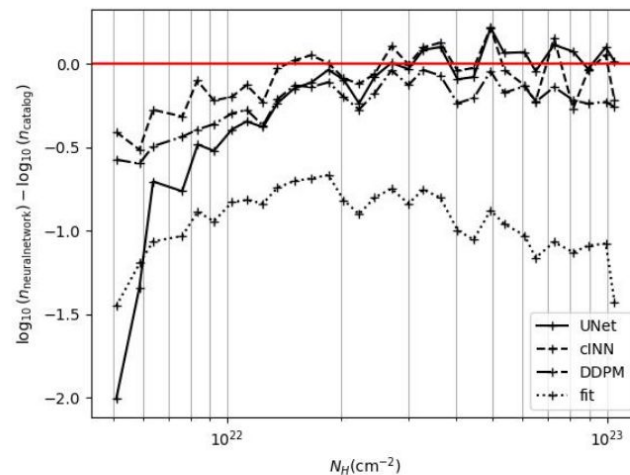
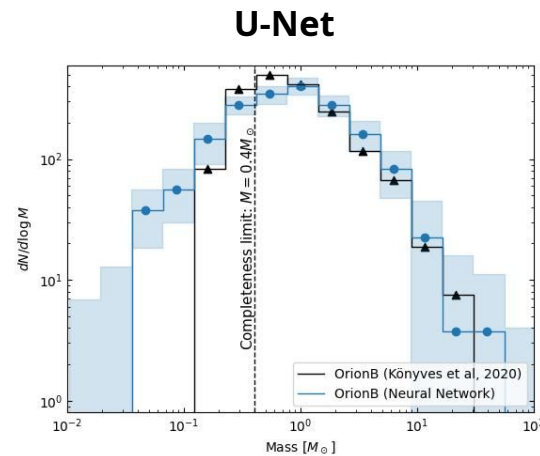
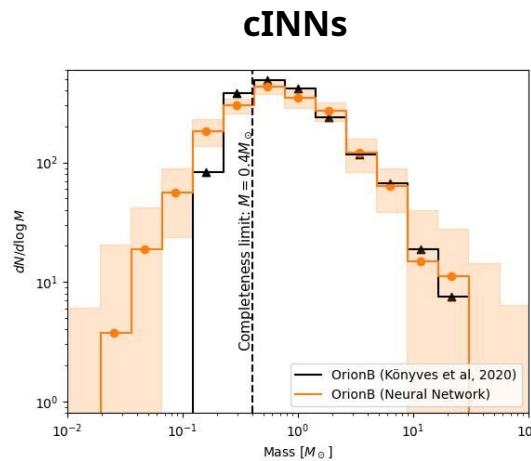
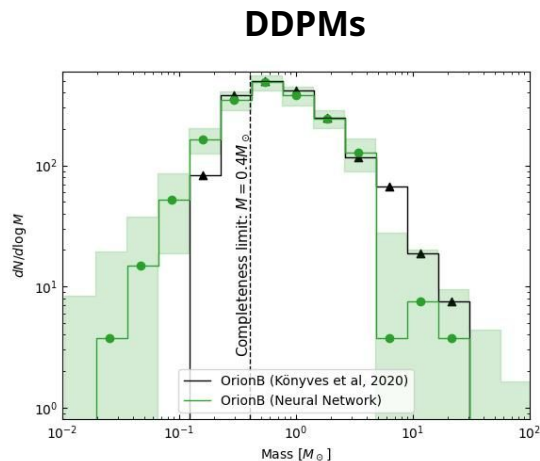


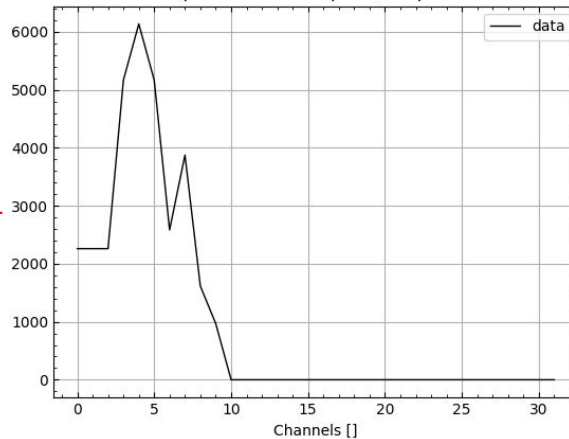
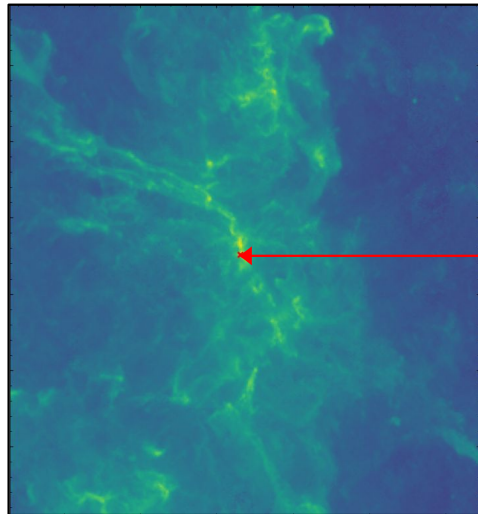
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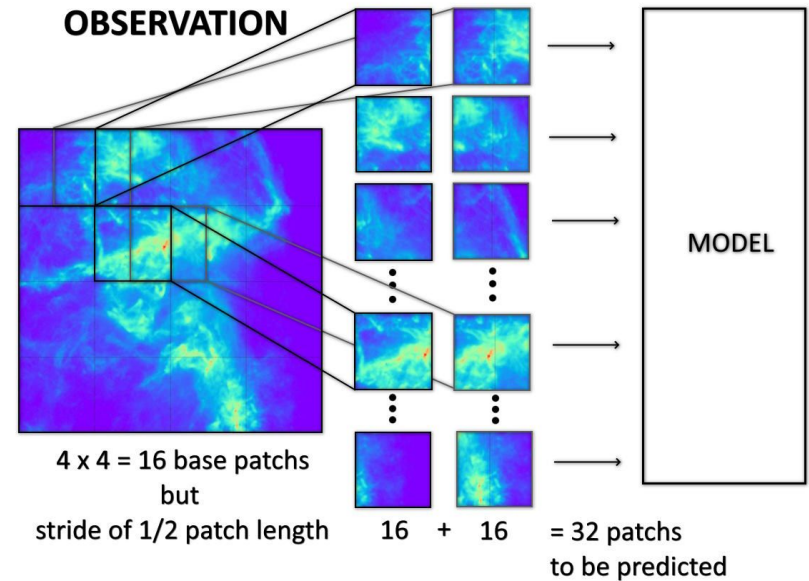
Color bars : 95 % confidence

Method to apply the neural networks on observations

- Neural network **applied repeatedly** to small observation **patches**
- Used for **probabilistic** modeling of line-of-sight densities
- Model predicts a **distribution** of average densities
- Outputs can be summarized using:
 - Mean predicted density
 - Most likely density value

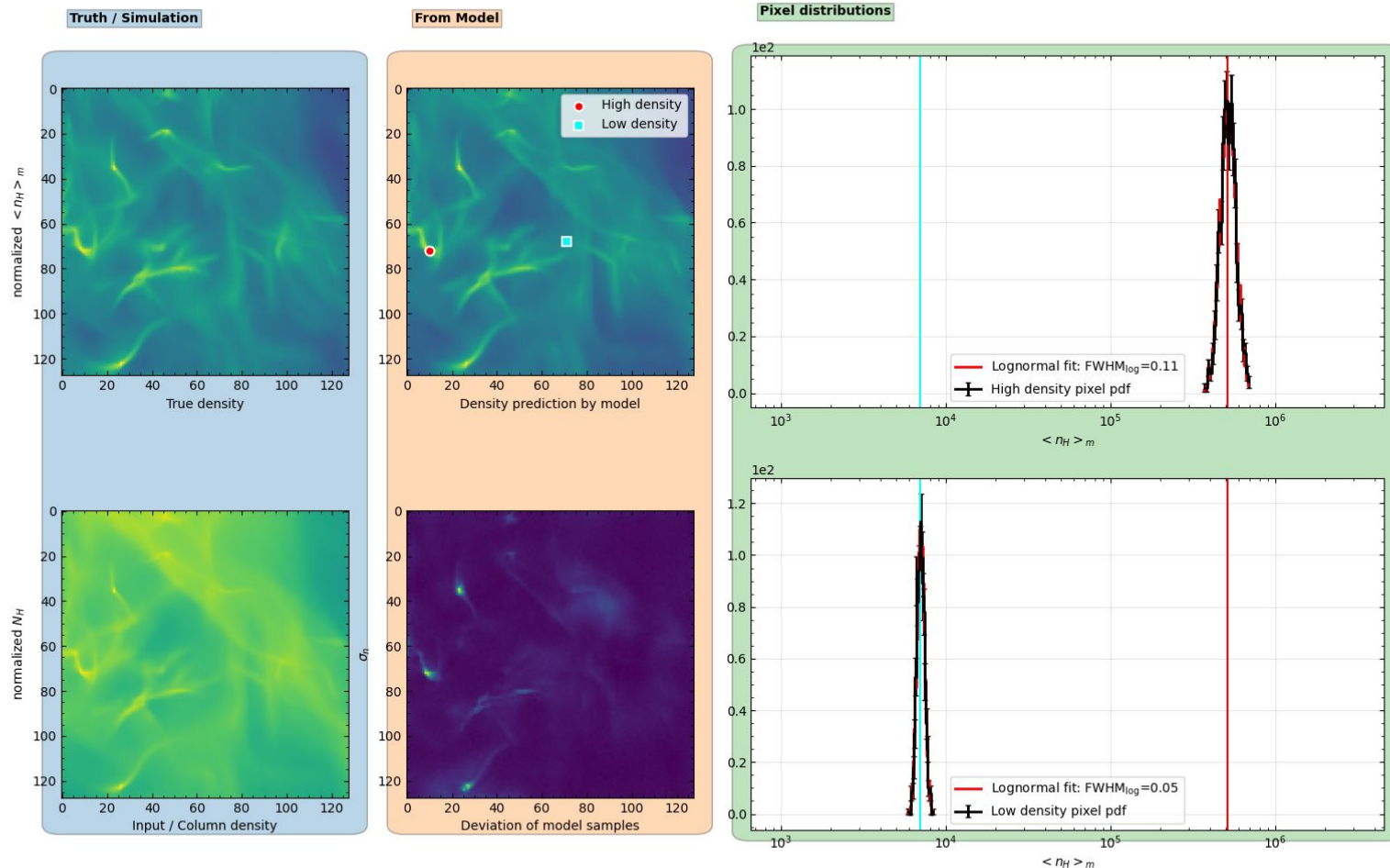


~Predicted mass-weighted average density along the l.o.s



Denoising Diffusion Probabilistic Models (DDPMs) - III

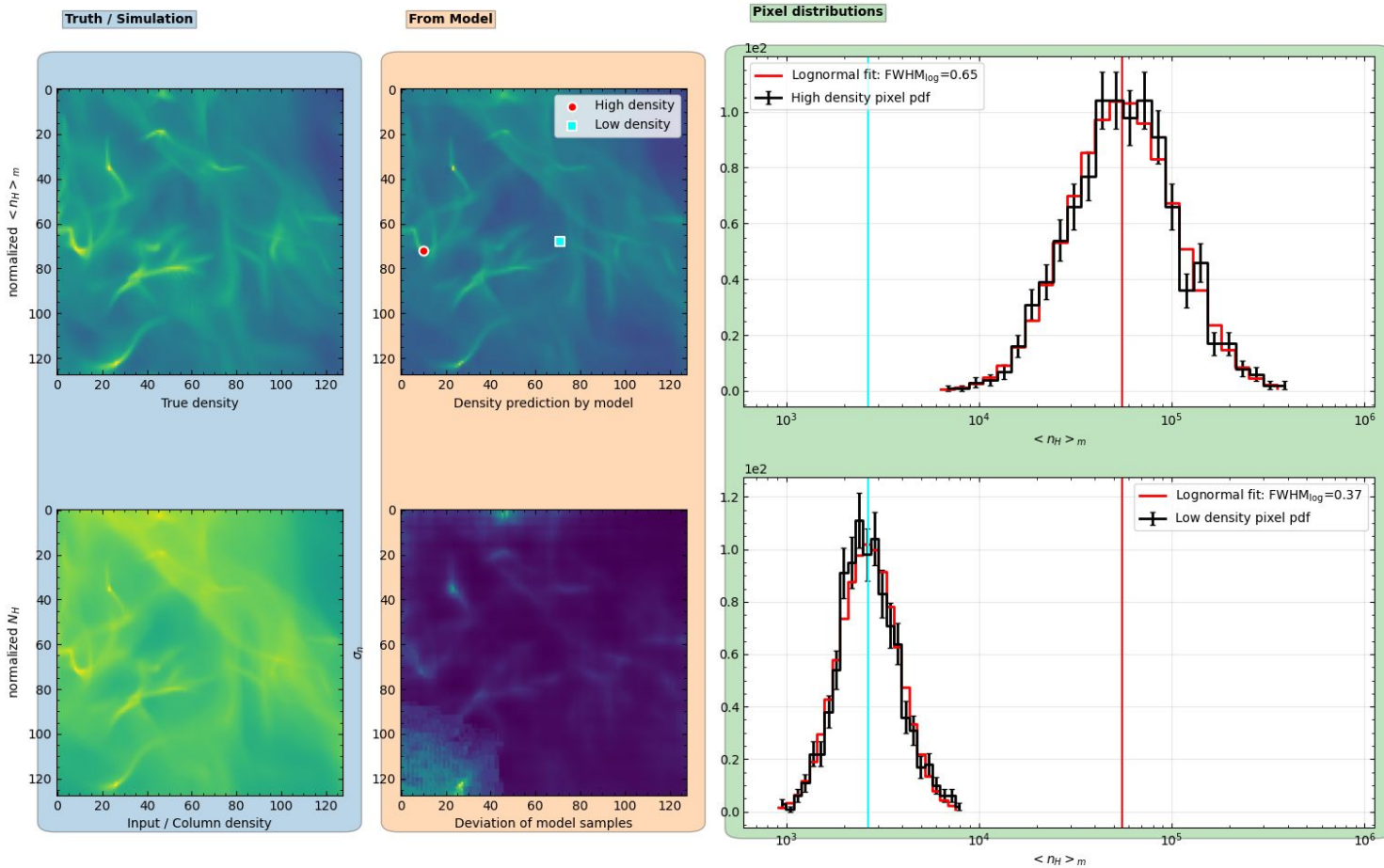
(DDPMs, Ho et al. (2020))



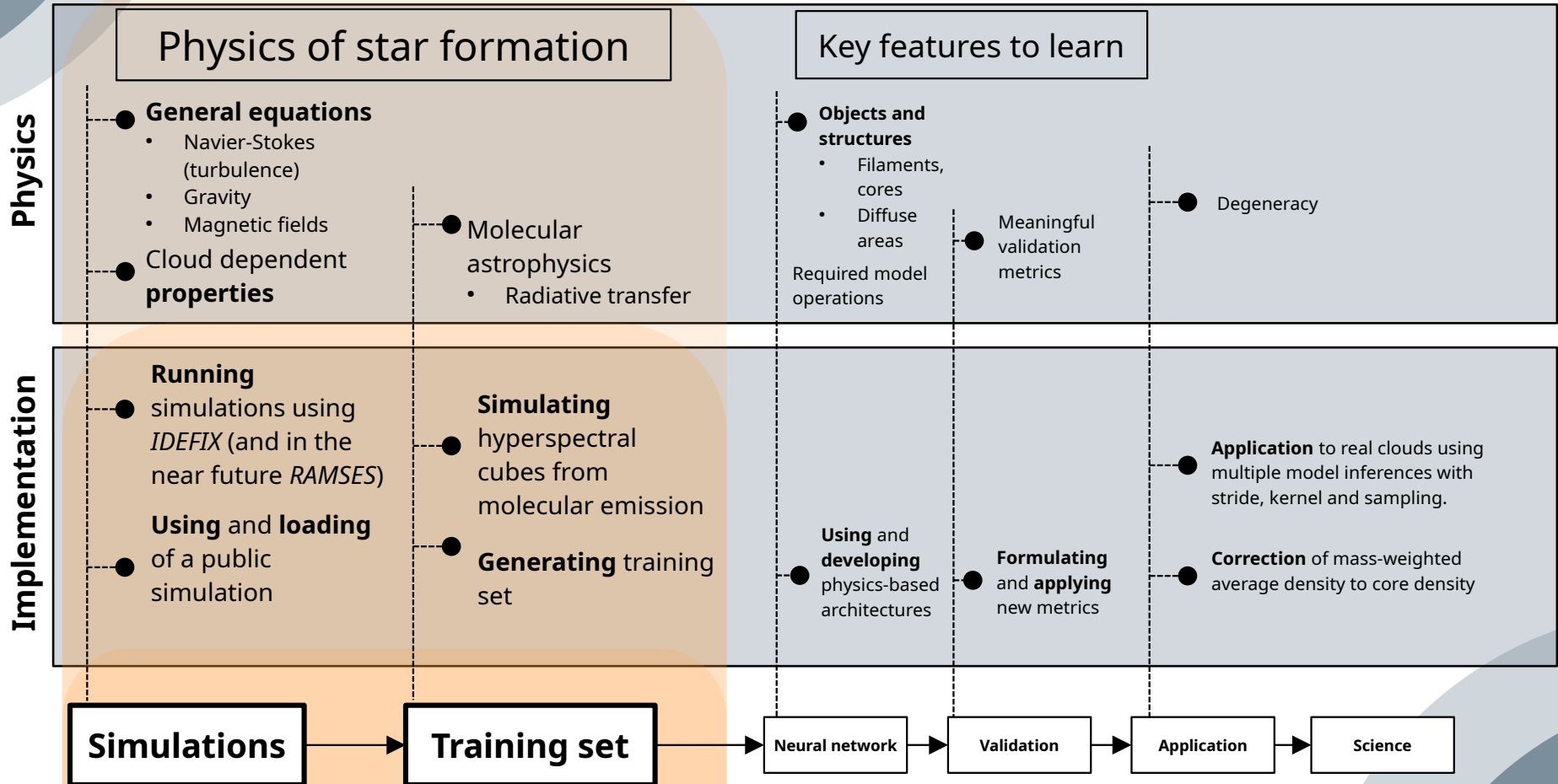


Conditional Invertible Neural Networks (cINNs) – II

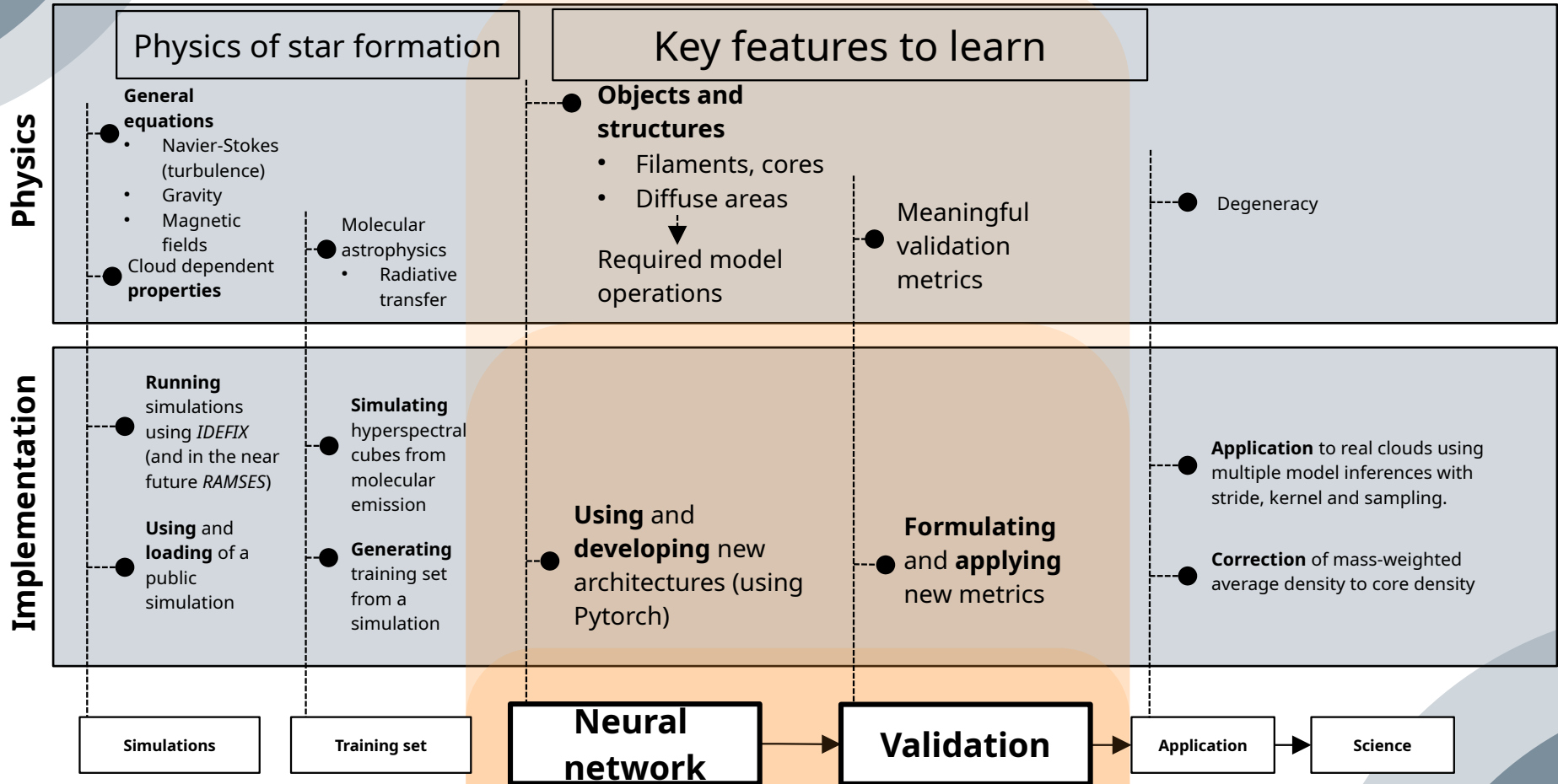
(cINNs, Ardizzone *et al.* (2021) ; Rezende & Mohamed (2016))



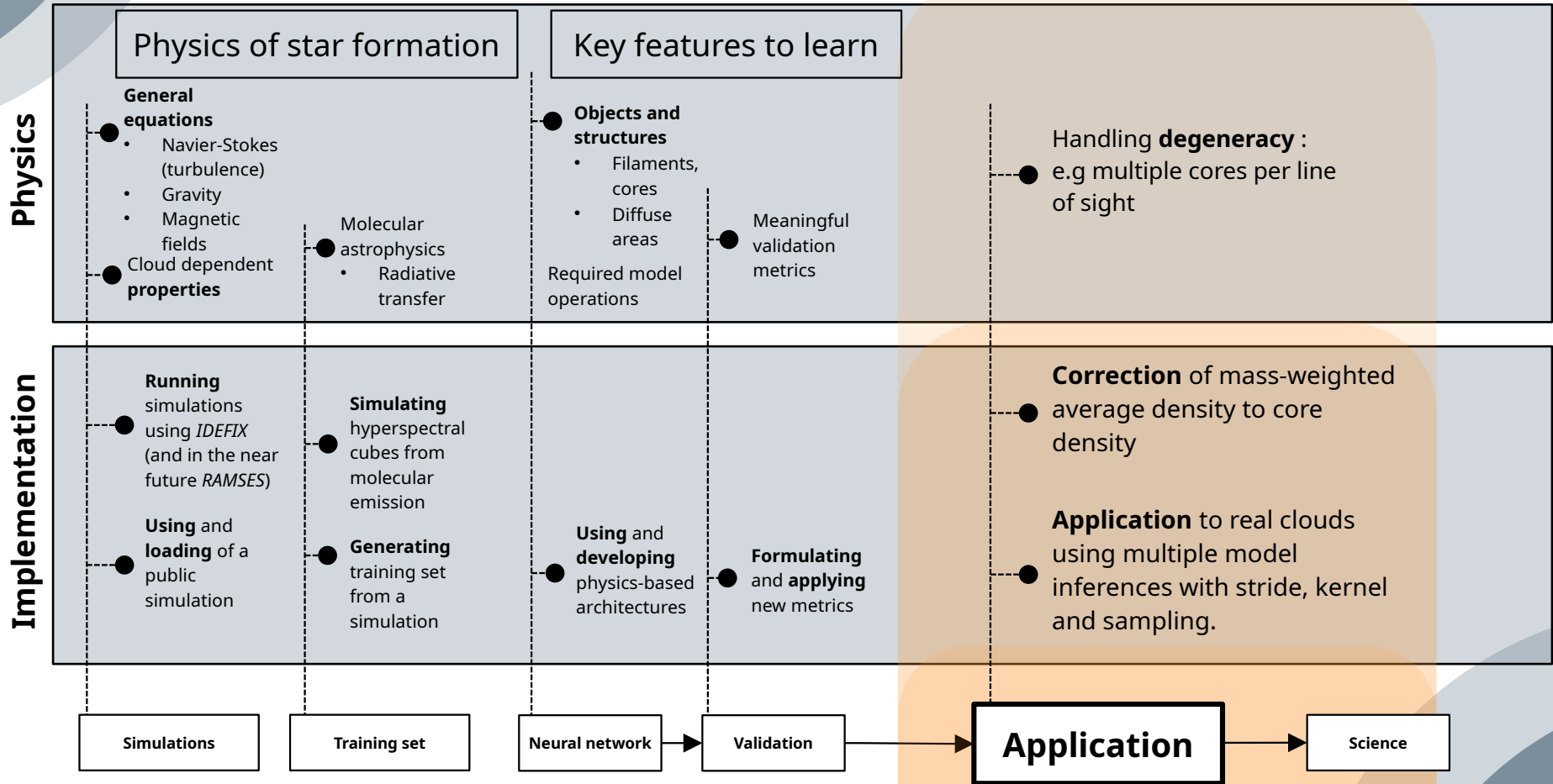
Pipeline - I



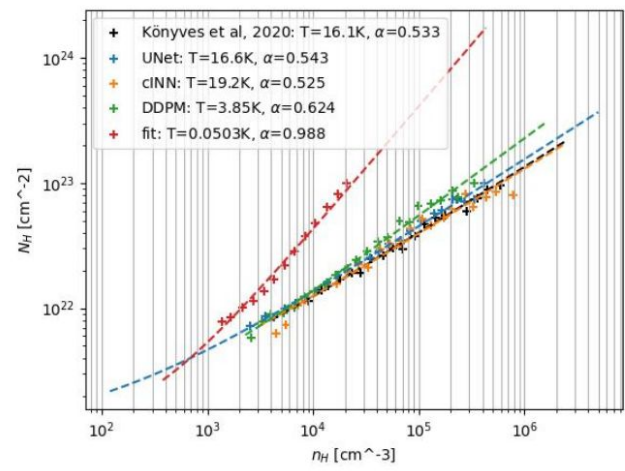
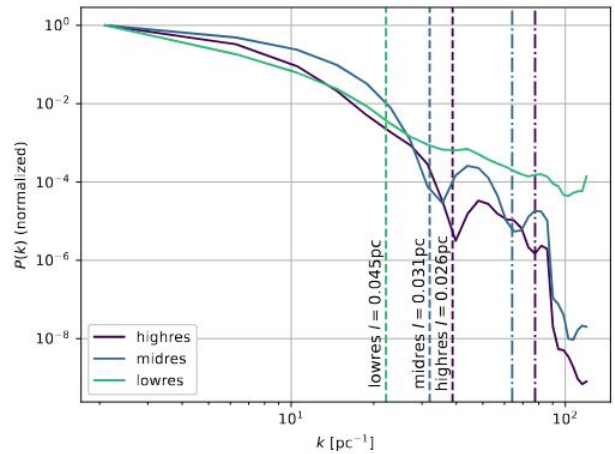
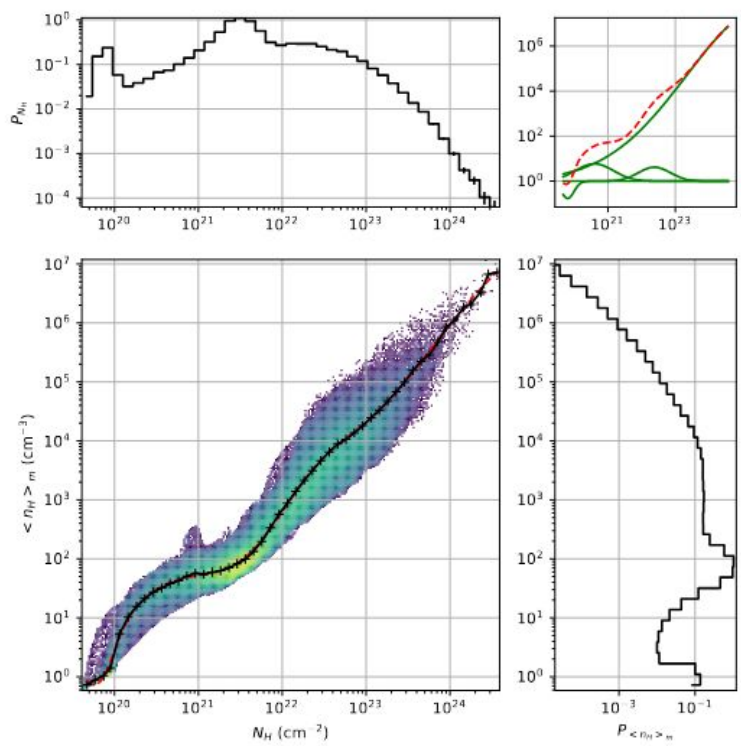
Pipeline - II



Pipeline - III



Extra figures



Correction

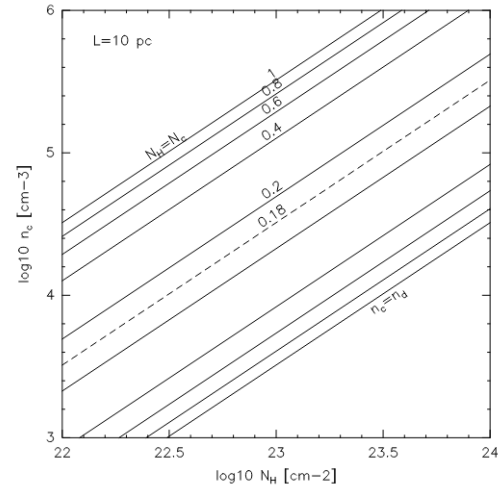
$$\frac{\langle n_{\text{H}} \rangle_{\text{m}}}{n_{\text{c}}} = \frac{n_{\text{d}}}{n_{\text{c}}} + \frac{n_{\text{c}} L_{\text{c}}}{N_{\text{H}}} \left(1 - \frac{n_{\text{d}}}{n_{\text{c}}} \right).$$

$$n_{\text{c}} = \frac{N_{\text{H}}}{L_{\text{c}} + L_{\text{d}}} \left\{ 1 + \left[1 - \left(\frac{L_{\text{d}}}{L_{\text{c}}} + 1 \right) \left(1 - \frac{\langle n_{\text{H}} \rangle_{\text{m}} L_{\text{d}}}{N_{\text{H}}} \right) \right]^{1/2} \right\}, \quad (\text{A.3})$$

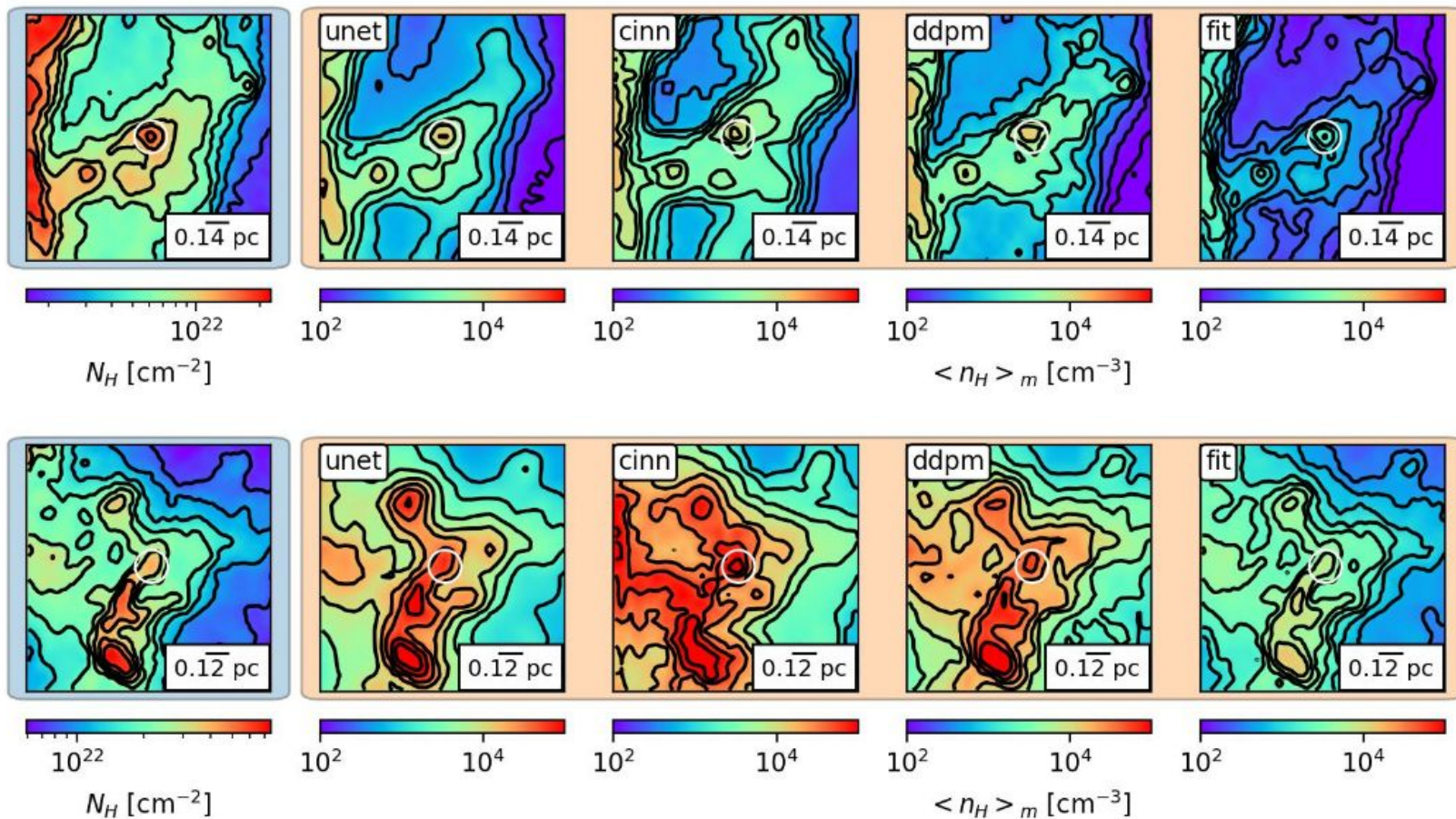
while if instead n_{d} is prescribed, then

$$n_{\text{c}} = \frac{n_{\text{d}}}{2} \left(1 + \sqrt{1 - \frac{4N_{\text{H}}}{L_{\text{c}} n_{\text{d}}} \left(1 - \frac{\langle n_{\text{H}} \rangle_{\text{m}}}{n_{\text{d}}} \right)} \right) \quad (\text{A.4})$$

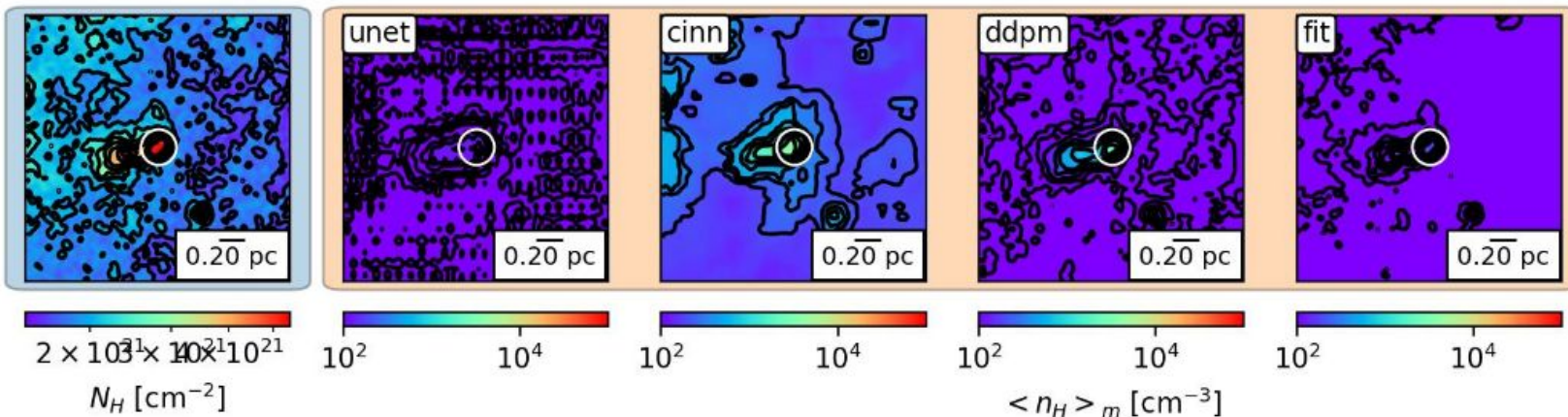
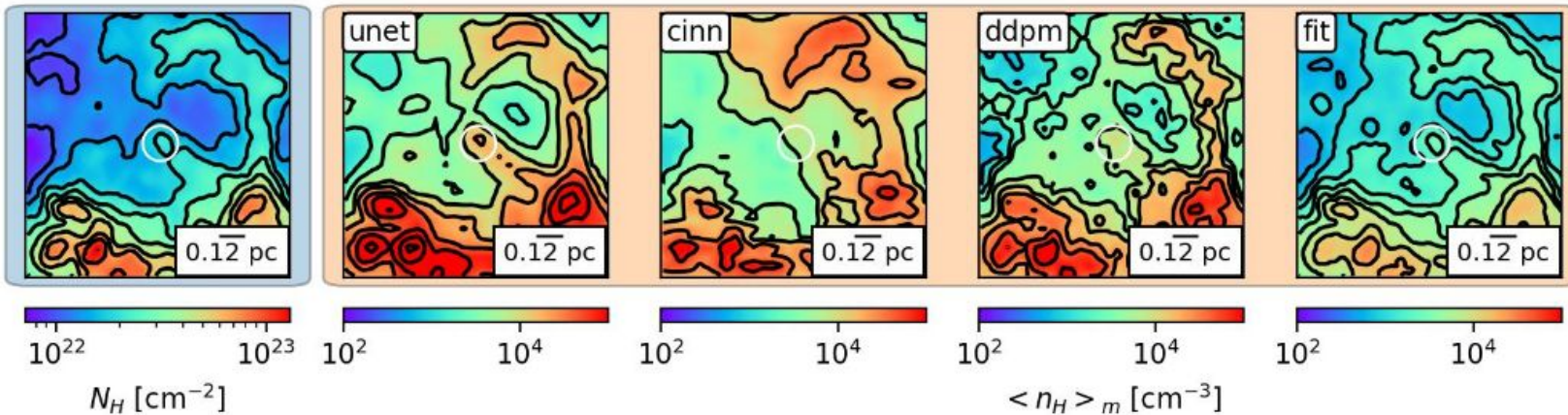
$$\lim_{L_{\text{d}} \rightarrow +\infty} n_{\text{c}} = \lim_{n_{\text{d}} \rightarrow 0} n_{\text{c}} = \sqrt{\frac{\langle n_{\text{H}} \rangle_{\text{m}} N_{\text{H}}}{L_{\text{c}}}},$$



Cores



Cores

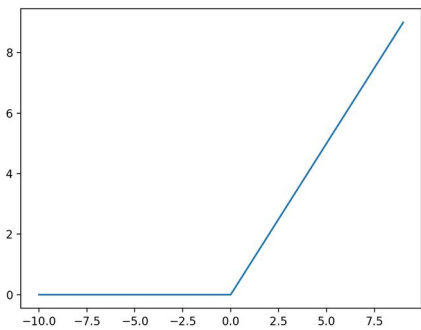
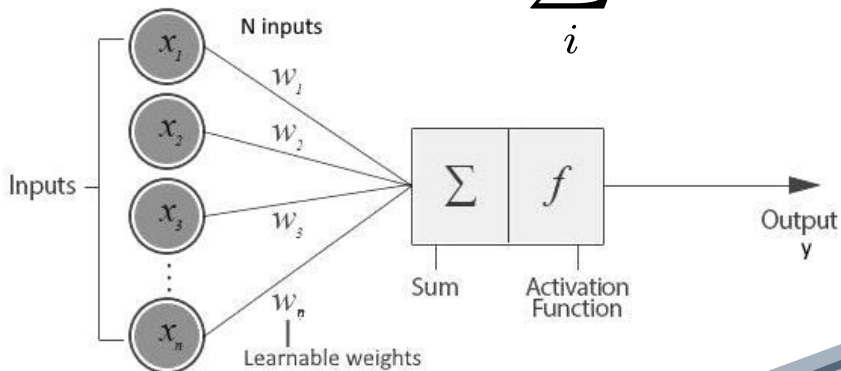




Neural networks

MLPs: Multi Layer perceptrons

$$y = f\left(\sum_i^N w_i x_i + b\right)$$



e.g. $f(x) = \text{ReLU} = \max(0, x)$

Input image



Convolution Kernel W

| | | |
|----------|----------|----------|
| w_{11} | w_{12} | w_{13} |
| w_{21} | w_{22} | w_{23} |
| w_{31} | w_{32} | w_{33} |

Feature map



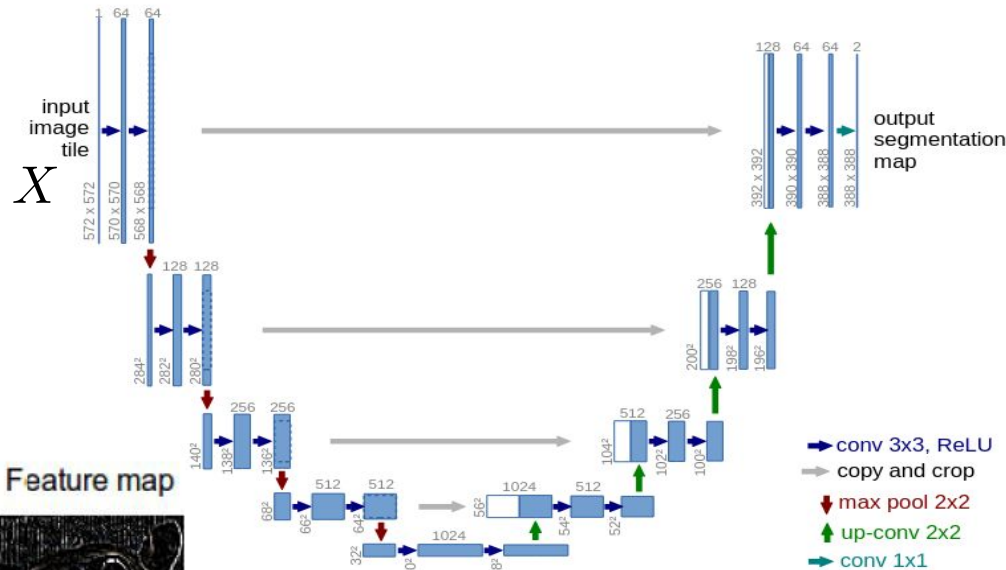
CNNs - UNet

$$Y_{i,j} = f\left(\sum_{m=1}^M \sum_{n=1}^N W_{m,n} \cdot X_{i+m,j+n} + b\right)$$

Pixel at (i,j) in the result image Y

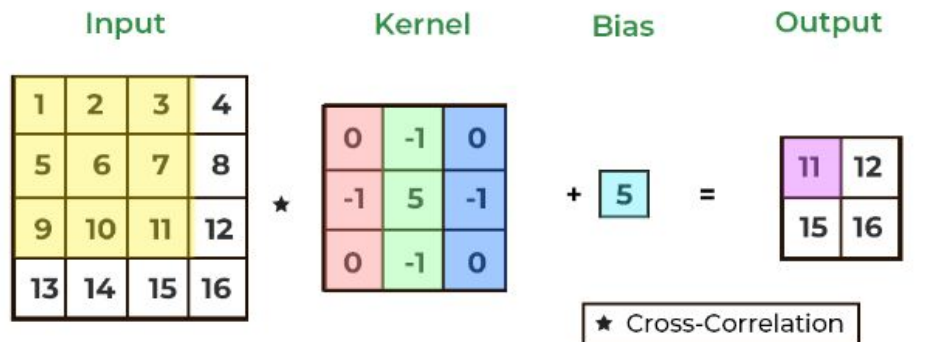
Element (m,n) of the kernel

Width and Height of the used kernel



[Ronneberger, O., Fischer, P., & Brox, T.; 2015]

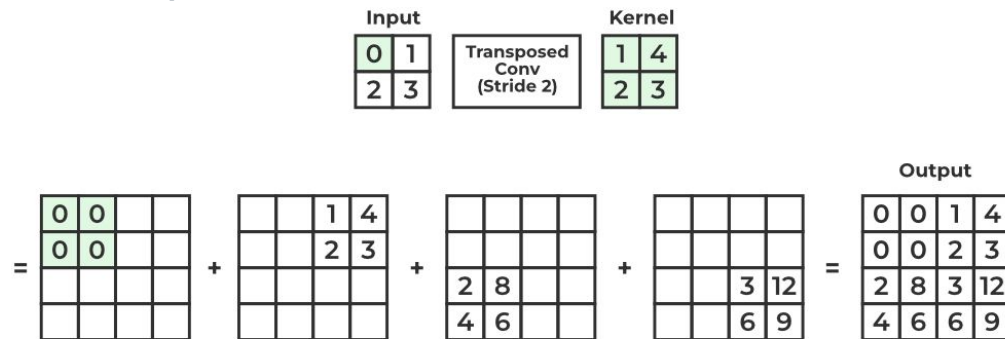
U-Net Operations



$$1 \times 0 + 5 \times -1 + 9 \times 0 + 2 \times -1 + 6 \times 5 + 10 \times -1 + 3 \times 0 + 7 \times -1 + 11 \times 0 + 5 = 11$$

Convolution

Transp Convolution



Max pooling (2x2)

